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CSAT
GS PAPER 2

Quantitative Aptitude, Logical Reasoning & Data Interpretation

UPSC CSAT PYQs
with
Topic-wise Segregation
2011-2023

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M.Sc. Mathematics
Indian Institute of Technology, Roorkee
Roorkee, India
(Founder and Director UNGIST Solution Pvt Ltd.)

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Preface

General Studies Paper II of the Civil Services Prelims Examination, also known as CSAT, has posed a significant challenge for many UPSC aspirants, particularly those without a mathematics background. Surprisingly, even engineering students have found it difficult to excel in this paper since 2021. The reason for this difficulty lies in the changing approach of UPSC, which now emphasizes logical questions rather than formula-based ones.

Teaching UPSC aspirants has always been a fascinating experience as they are diligent and adept at grasping logical concepts. I have consistently enjoyed relating these concepts to our everyday lives rather than relying on mathematical formulas. The classroom sessions have been fun and engaging. However, the lack of practice questions at the same difficulty level as the exam often made it challenging for students to solidify their understanding.

To address this gap, I was motivated to create this book that provides a comprehensive compilation of all the topics covered in the UPSC CSAT syllabus (excluding Reading Comprehension). The book contains theory and solved examples, along with unsolved practice questions for each topic. Most importantly, it includes UPSC CSAT questions from the past 13 years, enabling students to gauge the significance of each topic in the actual exam.

I extend my sincere gratitude to all my dedicated students who have inspired me to undertake this endeavor. I must mention some of them, such as Mr. Kapil Carpenter, Mr. Diptee Ranjan Nayak, and Dr. Pavani Reddy, who have been incredibly supportive throughout my writing journey.

I am also deeply thankful to Mr. Rajnish Gupta, whose efforts in converting my handwritten manuscript into a beautifully presented book have been invaluable. Additionally, I express my appreciation to Mr. Amitesh Kumar (AAKASH Printers) for his assistance in bringing this book to life.

Every effort has been made to ensure that this book is free of errors. However, should any discrepancies come to light, I humbly request my students and readers to notify me so that they can be rectified in the next edition.

I hope this book serves as a valuable resource to UPSC aspirants in their preparation for the CSAT examination.

Ram Mohan Pandey

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Quantitative Aptitude

Chapter 1 — Number System

1.1 Numbers

Basic Introduction

```
                          Complex Numbers
                          /            \
            Imaginary numbers          Real numbers
                                        /         \
                        Irrational numbers       Rational numbers
                                                  /           \
                                          Fractions          Integers
                                                             /        \
                                          Negative integers           Whole numbers
                                                          \                /        \
                                                         Zero {0}          Positive integers
                                                                           (Natural numbers)
```
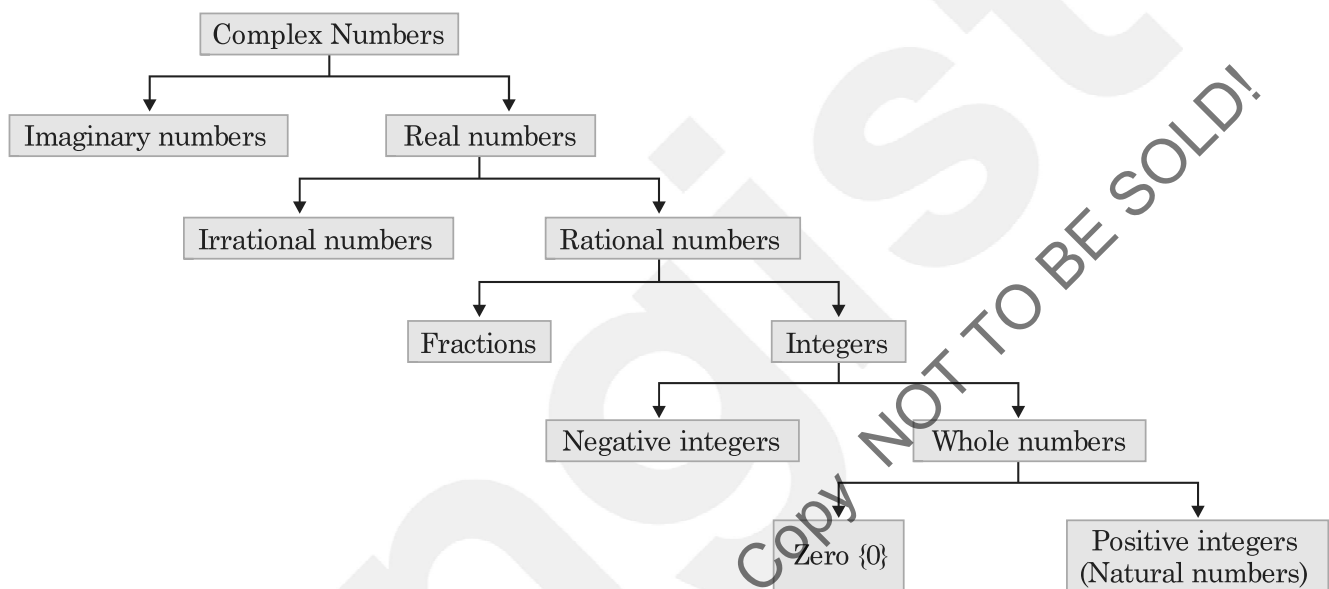
Prime number : A natural number which has only 2 factors *i.e.*, 1 and itself is called a prime number. *For example*: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, ... etc.

A prime number (>3) can always be expressed in the form of $6n \pm 1$ or $4n \pm 1$, where n is a natural number but the converse is *not* true.

Important facts about prime numbers :

a.   1 is neither prime nor composite number.

b.   2 is the only even prime number and the first prime number.

c.   There are infinite prime numbers.

d.   There are total 25 prime numbers up to 100.

e.   There are total 46 prime numbers up to 200.

List of prime numbers from 1 to 100 is given below :

2	11	23	31	41	53	61	71	83	97
3	13	29	37	43	59	67	73	89	
5	17			47			79		
7	19								

How to test whether a number is prime or not: (Root algorithm)

To test a number n, take the approximate square root of n and consider as it is, if it is a natural number otherwise just increase the square root of it to the next natural number. Then divide the given number by all the prime numbers below the square root obtained. If the number is divisible by any of these prime number, then it is not a prime number else it is a *prime number*.

Example: Check whether 241 is prime.

Solution: When we take the square root of 241, it is approximate more than 15, so we consider it 16. Now we divide 241 by all the prime numbers below 16 viz., 2, 3, 5, 7, 11, 13.

Since, 241 is not divisible by any one of the prime numbers below 16. So, it is a prime number.

Co-prime number: If HCF(a, b) = 1, then a and b are said to be co-prime to each other. *For example*, 8 and 15 are co-prime to each other.

Even number: An integer which is divisible by 2, is known as an even number. General form of an even number is $2n$; where n is an integer. Even number can be positive or negative as well. 0 is an even number.

Odd number: An integer which is not divisible by 2, is known as an odd number. General form of an odd number is $2n \pm 1$; where n is an integer.

We know the fact :

a.   Even + even = even

b.   Even + odd = odd

c.   Odd + odd = even

<div align="center">**EXAMPLES**</div>

1.   The sum of three prime numbers is 100. If one of them exceeds another by 24. Find the largest among them ?

Sol: Sum of three numbers is 100, which is an even number, if sum of three numbers is an even number, then either one of them is even or all three of them are even.

In the given condition all three of them cannot be even, since they have to be prime numbers.

So, one number will be even and two of them will be odd. We know 2 is the only even prime number.

Let the three number are a, b, c

According to the question,

$a + b + c = 100$ …(1)

Let $a = 2$

So, $b + c = 98$ …(2)

According to the questions,

$b - c = 24$

On solving these equation, we get

$b = 61$ and $c = 37$

So, the numbers are 2, 61 and 37.

Hence, 61 is the largest among them.

1.2 Summation 'n' (Σn), Σn^2 and Σn^3

1.2.1 Sum of first 'n' natural numbers is denoted by

$$\Sigma n = 1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$$

For example, $\Sigma 5 = 1 + 2 + 3 + 4 + 5 = \frac{5 \times 6}{2} = 15$.

1.2.2 Sum of the squares of the first 'n' natural numbers is denoted by

$$\Sigma n^2 = 1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

For example, $\Sigma 5^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 = \frac{5 \times 6 \times 11}{6} = 55$.

1.2.3 Sum of the cubes of the first 'n' natural numbers is denoted by

$$\Sigma n^3 = 1^3 + 2^3 + 3^3 + \ldots + n^3 = (\Sigma n)^2 = \left[\frac{n(n+1)}{2}\right]^2$$

For example, $\Sigma 5^3 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 = (\Sigma n)^2 = \left[\frac{5 \times 6}{2}\right]^2 = 15^2 = 225$.

1.2.4 Sum of first 'n' odd natural numbers = n^2.

For example, $1 + 3 + 5 = 9 = 3^2$. $1 + 3 + 5 + 7 = 16 = 4^2$. $1 + 3 + 5 + 7 + 9 = 25 = 5^2$.

EXAMPLES

1. If natural number are arranged row-wise in the following manner, what will be last term of 10th row (R_{10}) ?

$R_1 = 1$
$R_2 = 2, 3$
$R_3 = 4, 5, 6$
$R_4 = 7, 8, 9, 10$

Sol: If we observe last term of every row carefully, we can say,

Last term of $R_1 = \Sigma 1 = 1$
Last term of $R_2 = \Sigma 2 = 1 + 2 = 3$
Last term of $R_3 = \Sigma 3 = 1 + 2 + 3 = 6$
Last term of $R_4 = \Sigma 4 = 1 + 2 + 3 + 4 = 10$

So, last term of $R_{10} = \Sigma 10 = 1 + 2 + 3 + \ldots + 10 = \frac{10 \times (10+1)}{2} = 55$.

2. Consider the sequence of ordered sets of natural numbers: [1], [2, 3], [4, 5, 6], … What is the last number in the 10th set ?

Sol: This is very much similar to example 1. The answer is 55.

3. What will be 200th term of the given sequence 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, … ?

Sol: In the given sequence we can observe, 1st term = 1; 3rd term = 2; 6th term = 3; 10th term = 4

We know that, $\Sigma 1 = 1$, $\Sigma 2 = 3$, $\Sigma 3 = 6$, $\Sigma 4 = 10$

So, we can say $(\Sigma n)^{th}$ term will be 'n' and with 'n' is written last time in this sequence.

Therefore, we need the check summation of which number is closed to 200.

10. If $1^3 + 2^3 + 3^3 + 4^3 + ... + 10^3 = 3025$, then what will be the value of $2^3 + 4^3 + 6^3 + 8^3 + ... + 20^3$?

Sol: $2^3 + 4^3 + 6^3 + 8^3 + ... + 20^3 = 2^3 (1^3 + 2^3 + 3^3 + 4^3 + ... + 10^3) = 8 \times 3025 = 24200$.

1.3 $xy - yx = 9|x - y|$, $xyz - zyx = 99|x - z|$, $xy + yx = 11(x + y)$

1.3.1 The difference between a 2-digit number 'xy' and the number obtained by interchanging the positions of its digits 'yx' is always divisible by 9 and equals $9|x - y|$.

    Proof: $xy - yx = 10x + y - (10y + x)$

$$= 10x + y - 10y - x$$
$$= 9(x - y)$$

    To make the difference always positive, we use the modules.

    So, we can say $xy - yx = 9|x - y|$.

1.3.2 Difference between a 3-digit number 'xyz' and its reverse 'zyx' is always a multiple of 99, and equals $99|x - z|$.

    The difference between a 3-digit number 'xyz' and the number obtained by interchanging the positions of first and last digits 'zyx' is always divisible by which number.

    Proof: We know, $xyz = 100x + 10y + z$ and $zyx = 100z + 10y + x$

    So, $xyz - zyx = (100x + 10y + z) - (100z + 10y + x) = 99(x - z)$

    To make the difference positive, we use modulus so, $xyz - zyx = 99|x - z|$

    Hence, we can say the difference between a 3-digit number 'xyz' and the number obtained by interchanging the positions of first and last digits 'zyx' is always divisible 99.

1.3.3 Sum of a 2-digit number 'xy' and its reverse 'yx' is always a multiple of 11, and equals $11(x + y)$.

    Proof: $xy + yx = 10x + y + 10y + x$

$$= 11x + 11y = 11(x + y)$$

EXAMPLES

1. If 54 is added to a 2-digit number, its digits get reversed. How many such 2-digit numbers are possible ?

Sol: According to the question; $xy + 54 = yx$

    Or, $xy - yx = 54$

    Or, $9|x - y| = 54$ (since, $xy - yx = 9|x - y|$)

    So, $|x - y| = 6$.

    Difference of digits will be 6.

    Hence, possible values of 'xy' will be 17, 28 and 39.

    So, 3 such numbers are there *i.e.,* 17, 28 and 39.

2. If we add 36 to a 2-digit number, its digits get reversed. How many such 2-digit numbers are possible ?

Sol: According to the question; $xy + 36 = yx$

    Or, $xy - yx = 36$

    Or, $9|x - y| = 36$

    So, $|x - y| = 4$.

    Difference of digits will be 4.

    Hence, possible values of 'xy' will be 15, 26, 37, 48, 59.

    So, 5 such numbers are there.

1.4 Cyclicity (unit digit)

Cyclicity of 2

We all know,

$$2^1 = \boxed{2} \qquad 2^5 = 3\boxed{2} \qquad 2^9 = 51\boxed{2}$$
$$2^2 = \boxed{4} \qquad 2^6 = 6\boxed{4} \qquad 2^{10} = 102\boxed{4}$$
$$2^3 = \boxed{8} \qquad 2^7 = 12\boxed{8} \qquad 2^{11} = 204\boxed{8}$$
$$2^4 = 1\boxed{6} \qquad 2^8 = 25\boxed{6} \qquad 2^{12} = 409\boxed{6}$$

We can observe that the unit digit gets repeated after every 4th power of 2. It is actually a cycle of 2, 4, 8, 6 which will get repeated all the times. Hence, we can say cyclicity of 2 is 4.

This means that, a number of the form

2^{4k+1} will have the last digit as 2.

2^{4k+2} will have the last digit as 4.

2^{4k+3} will have the last digit as 8.

2^{4k} will have the last digit as 6 (where k = 1, 2, 3, …).

This is applicable not only for 2, but for all numbers ending in 2. Which can be concluded in the following table:

Form of power	Unit digit
$4n + 1$	2
$4n + 2$	4
$4n + 3$	8
$4n$	6

Similarly, we can find the cyclicity of other digit's too. Let's talk about the cyclicity of 3.

Cyclicity of 3

$$3^1 = \boxed{3} \qquad 3^5 = 24\boxed{3}$$
$$3^2 = \boxed{9} \qquad 3^6 = 72\boxed{9}$$
$$3^3 = 2\boxed{7} \qquad 3^7 = 218\boxed{7}$$
$$3^4 = 8\boxed{1} \qquad 3^8 = 656\boxed{1}$$

Hence, cyclicity of 3 = 4.

Form of power	Unit digit
$4n + 1$	3
$4n + 2$	9
$4n + 3$	7
$4n$	1

Cyclicity of 4

$$4^1 = \boxed{4} \qquad 4^3 = 6\boxed{4} \qquad 4^5 = 102\boxed{4}$$
$$4^2 = 1\boxed{6} \qquad 4^4 = 25\boxed{6} \qquad 4^6 = 409\boxed{6}$$

Hence, cyclicity of 4 = 2.

We can say $4^{\text{odd number}}$ has 4 as its unit digit and $4^{\text{even number}}$ has 6 as its unit digit.

6. What will be the unit digit of the expression 522^{522} ?

Sol: Unit digit of 522^{522} is same as 2^{522} and unit digit of $2^{522} = 4$ (since, 522 is of the form $4n + 2$)

Hence, unit digit of 522^{522} is 4.

7. Find the unit digit of the given sum $1! + 2! + 3! + \ldots + 100!$. (where, $n! = 1 \times 2 \times 3 \times \ldots \times n$)

Sol: We know that, $1! = 1$, $2! = 2$, $3! = 6$, $4! = 24$, $5! = 120$, $6! = 720$, $7! = 5040$ and so on.

Here, we observe from 5! to 100!, all the values have 0 as their unit's digit, so if we add from 5! to 100! there will be 0 as the unit digit in the resultant and if we add from 1! to 4!, we have 3 as unit's place (since $4! + 3! + 2! + 1! = 24 + 6 + 2 + 1 = 33$).

So finally, we add $1! + 2! + 3! + \ldots + 100!$, we have 3 as the unit digit of the resultant.

Note: If 0! is also added, the unit's digit would be $3 + 1 = 4$. (since, $0! = 1$).

8. When 2^{33} is divided by 10, the remainder will be

Sol: If we divide a number by 10; we get unit digit as remainder.

Here, this question is trying to ask the unit digit of 2^{33}, which is 2.

9. What is the unit digit of $(2017)^{2017}$?

Sol: Unit digit of $(2017)^{2017} \equiv$ unit digit of $7^{2017} \equiv 7^{4n+1} \equiv 7^1 = 7$.

Since, cyclicity of 7 is 4. So, unit digit of 7^{4n+1} is 7.

10. Consider the set of numbers $\{17^1, 17^2, 17^3, \ldots, 17^{300}\}$. How many of these numbers end with the digit 3 ?

Sol: We know the power cycle of 7 as:

$7^1 = 7$; $7^2 = 49$; $7^3 = 343$; $7^4 = 2401$; $7^5 = 16807$ and $7^6 = 117649$.

As it can be observed, the unit digit gets repeated after every 4th power of 7.

Hence, we can say that 7 has a power cycle of 7, 9, 3, 1 with frequency 4.

Number of sets of four numbers $\dfrac{300}{4} = 75$ sets.

1.5 Exponents (Number of zeros)

If we need to find the maximum power of 2, that will completely divide 10!. Then we have to expand 10! and need to find how many times 2 is getting multiplied with 2.

For example,

$10! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10$

$$
\begin{array}{ccccc}
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\
2\times1 & 2\times2 & 2\times3 & 2\times2\times2 & 2\times5 \\
1 & 2 & 1 & 3 & 1
\end{array}
$$

Total number of 2's $= 1 + 2 + 1 + 3 + 1 = 8$

So, 2^8 will divide 10! completely.

This method is feasible for smaller numbers like 10, 15, 20, etc. but for bigger numbers like 500, 700 or so on, this approach will not work. Hence, we have the following method to find total number of powers.

Algorithm

We can find the maximum power by successive division, till the division is possible.

Step 1: 10 will be divided by 2, which gives 5.

Step 2: 5 will be divided by 2, which gives 2 (greatest integer).

Step 3: 2 will be divided by 2, which gives 1. (we will stop here, since 1 is less than 2)

5. If N is the product of first 100 multiples of 10. Then find the number of zeros at the end of N?

Sol: According to the question,

$N = 10 \times 1 \times 10 \times 2 \times 10 \times 3 \times \ldots \ldots \times 10 \times 100$

$\quad = 10^{100}(1 \times 2 \times 3 \times \ldots \ldots \times 100) = 10^{100}(100!) = 100$ zeros $+ 24$ zeros $= 120$ zeros.

6. If $n!$ ends with 14 zeros, find the sum of all possible values of n?

Sol: This question should be solved by hit and trial method.

Where we need to think about first such number whose factorial is ending with 14 zeros; and next 4 numbers will also follow the same thing.

60! ends with 14 zeros. So, 60!, 61!, 62!, 63!, 64! are ending 14 zeros.

So, required sum $= 60 + 61 + 62 + 63 + 64 = 310$.

7. If $n!$ ends with 30 zeros, how many values of n are possible?

Sol: 124! is ending with 28 zeros while 125! is ending with 31 zeros.

So, there is no such number whose factorial is ending with 30 zeros.

1.6 Divisibility

Rules of divisibility by certain integers :

Divisibility by 2
A number is divisible by 2, if the last digit is even *i.e.,* 2, 4, 6, 8, or 0.
For example, 124 is divisible by 2 because it is even (it ends in a 4).

Divisibility by 3
If the sum of the digits is divisible by 3, then the number is divisible by 3.
For example, 129 is divisible by 3 because $1 + 2 + 9 = 12$, and 12 is divisible by 3.

Divisibility by 4
If the last 2-digits of a whole number are divisible by 4, then the entire number is divisible by 4.
For example, 1240 is divisible by 4 because 40 is divisible by 4.

Divisibility by 5
A number is divisible by 5 if and only if the last digit is 0 or 5.
For example, 150 is divisible by 5 because it ends with 0.

Divisibility by 6
If the number is divisible by 2 and 3, it is also divisible by 6.
For example, 1290 is divisible by 6 because it is divisible by 2 and 3 simultaneously.

Divisibility by 8
If the last 3-digits of a whole number are divisible by 8, then the entire number is divisible by 8.
For example, 1240 is divisible by 8 because 240 is divisible by 8 ($240 \div 8 = 30$).

Divisibility by 9
If the sum of the digits is divisible by 9, then the number is divisible by 9.
For example, 504 is divisible by 9 because $5 + 0 + 4 = 9$, and 9 is divisible by 9 ($9 \div 9 = 1$).

1.7   Remainder theorem

Binomial expansion is given by $(a+b)^n = n_{c_0} a^n b^0 + n_{c_1} a^{n-1} b + n_{c_2} a^{n-2} b^2 + \ldots + n_{c_n} a^0 b^n$

If $(a+b)^n$ is divided by a, we will be having following expression.

$$\frac{(a+b)^n}{a} = \frac{n_{c_0} a^n b^0}{a} + \frac{n_{c_1} a^{n-1} b}{a} + \frac{n_{c_2} a^{n-2} b^2}{a} + \ldots + \frac{n_{c_n} a^0 b^n}{a}$$

Here in RHS side, remainder of last term will be having some significant value, rest will have the remainders as 0. Because in all other term we have a multiple of 'a' in numerator.

$$\frac{(a+b)^n}{a} = 0 + 0 + 0 + \ldots + \frac{b^n}{a}$$

Hence, while solving the questions we will follow the same approach.

For example, $\dfrac{4^{98}}{3} = \dfrac{(3+1)^{98}}{3} = 0 + 0 + 0 + \ldots + \dfrac{1^{98}}{3}$

Numerator can be expanded with the help of binomial expansion and the last (final) term of numerator will be $1^{98} = 1$.

And when 1 is divided by 3, remainder will be 1.

So, 4^{98} will give 1 as remainder, when it is divided by 3.

Some important results :

1.   The remainder of the expression, $\dfrac{4^n}{6}$ (where, n is a natural number) is always 4.

2.   Wilson's theorem

A positive integer $n > 1$ is a prime if and only if $(n-1)! \equiv -1 \pmod{n}$.

For example :

$\dfrac{4!}{5}$ will give 4 as remainder. $\dfrac{6!}{7}$ will give 6 as remainder.

$\dfrac{52!}{53}$ will give 52 as remainder.   **[GATE 2017]**

EXAMPLES

1.   What will be the remainder of the expression $\dfrac{13^{90}}{3}$?

Sol: $\dfrac{13^{90}}{3} = \dfrac{(12+1)^{90}}{3} \equiv \dfrac{1^{90}}{3} = 1$ (Remainder)

We should break the numerator at very closed to the greatest multiple of denominator, so that we can get 0 as remainders in all the terms except the last term.

2.   If 2^{200} is divided by 17, then what will be the remainder ?

Sol: $\dfrac{2^{200}}{17} = \dfrac{(2^4)^{50}}{17} \equiv \dfrac{16^{50}}{17} = \dfrac{(17-1)^{50}}{17} \equiv \dfrac{(-1)^{50}}{17} = \dfrac{1}{17} = 1$ (Remainder)

Here, we need to check, which power of 2 is closed to 17.

So, we are getting 2^4.

12. What will be the remainder of the expression $\dfrac{3^0 + 3^1 + 3^2 + 3^3 + \cdots\cdots + 3^{79}}{13}$?

Sol: Here the pattern will be 1 + 3 + 9, 1 + 3 + 9, 1 + 3 + 9, … and we have total 80 terms in numerator. So, sum of 79th and 80th term will be final remainder.

Hence, final remainder will be 1 + 3 = 4.

1.8   LCM and HCF

1. Factors and Multiples : If a number a divides another number b exactly, we say that a is a *factor* of b. In this case, b is called a *multiple* of a.

2. Highest Common Factor (HCF) : The HCF of two or more than two numbers is the greatest number that divides each of them exactly.
    To find HCF: Express each one of the given numbers as the product of prime factors. The product of least powers of common prime factors gives HCF.

3. Least Common Multiple (LCM) : The least number which is exactly divisible by each one of the given numbers is called their LCM.
    To find LCM: Break each one of the given numbers into a product of prime factors. Then LCM is the product of highest powers of all the factors.

4. Product of two numbers = Product of their HCF and LCM (valid only for 2 numbers).

5. HCF is always a factor of LCM.

6. LCM is always a multiple of HCF.

7. LCM of fractions $= \dfrac{\text{LCM of numerators}}{\text{HCF of denominators}}$

8. HCF of fractions $= \dfrac{\text{HCF of numerators}}{\text{LCM of denominators}}$

EXAMPLES

1. Find the HCF and LCM of 28, 54, 88.

Sol: We have, $28 = 2^2 \times 7$

$54 = 2 \times 3^3$

$88 = 2^3 \times 11$

So, HCF of (28, 54, 88) = 2 and LCM $= 2^3 \times 3^3 \times 7 \times 11$.

2. Find out the HCF of 210 and 150.

Sol: $210 = 2 \times 3 \times 5 \times 7$

$150 = 2 \times 3 \times 5^2$

LCM $= 2 \times 3 \times 5^2 \times 7 = 1050$.

3. Find out the HCF and LCM of $2^3 \times 3^4 \times 5^2 \times 7$ and $2^2 \times 3^2 \times 7^2$.

Sol: HCF $= 2^2 \times 3^2 \times 7$

LCM $= 2^3 \times 3^4 \times 5^2 \times 7^2$.

1.9 Factors

Factors are the numbers that can divide a number exactly. Hence, after division, there is no remainder left.

To find the total number of factors of N.

Let us assume N is a natural number, for which we need to find the factors. If we convert N into the product of prime numbers by prime factorization method, we can represent it as :

$N = x^a \times y^b \times z^c$

where x, y and z are the prime numbers and a, b and c are their respective powers.

Total number of factors of a number $N = (a + 1)(b + 1)(c + 1)$.

EXAMPLES

1. Find the total number of factors of 360.

Sol: $360 = 2^3 \times 3^2 \times 5$

Total number of factors of 360 = (3 + 1)(2 + 1)(1 + 1) = 4 × 3 × 2 = 24.

2. Find the total number of factors of 5000.

Sol: $5000 = 2^3 \times 5^4$

Total number of factors of 5000 = (3 + 1)(4 + 1) = 4 × 5 = 20.

3. Find the total number of odd factors of 360.

Sol: To find the odd factors, ignore the powers of 2 and find the total number of factors.

To find all the odd factors of 360, we need to find the total factors of $3^2 \times 5$.

$360 = 2^3 \times 3^2 \times 5$ here, $3^2 \times 5$ is greatest odd factor of 360.

Total factors of $3^2 \times 5$ = (2 + 1)(1 + 1) = 3 × 2 = 6.

Hence, 360 has 6 odd factors.

4. Find the total number of even factors of 1000.

Sol: $1000 = 2^3 \times 5^3$

Greatest odd factor of 1000 = 5^3.

Total odd factors of 1000 = Total factors of 5^3 = 3 + 1.

Total factors of 1000 = $2^3 \times 5^3$ = (3 + 1)(3 + 1) = 4 × 4 = 16

So, total even factors of 1000 = Total factors – odd factors = 16 – 4 = 12.

5. How many factors of 400 are perfect squares ?

Sol: 400 = 20 × 20. We need to find total factors of 20, and square of these numbers will be factors of 400 and they are perfect square also. So, factors of 20 = 1, 2, 4, 5, 10, 20.

Hence, 1^2, 2^2, 4^2, 5^2, 10^2, 20^2 *i.e.,* numbers, which factors are 400 and perfect squares also.

6. How many factors of 800 are perfect squares ?

Sol: 800 = 400 × 2. Here, we need to find the largest factor of given number which is perfect square itself and solve like previous example.

Practice Set

Numbers

1. $2^{73} - 2^{72} - 2^{71}$ is the same as
   (a) 0   (b) 2^{70}
   (c) 2^{71} (d) 2^{72}

2. A positive integer is said to be a prime number if it is not divisible by any positive integer other than itself and 1. Let p be a prime number greater than 5, then $(p^2 - 1)$ is
   (a) never divisible by 6.
   (b) always divisible by 6, and may or may not be divisible by 12.
   (c) always divisible by 12, and may or may not be divisible by 24.
   (d) always divisible by 24.

3. The sum of two numbers is equal to thrice their difference. If the smaller of the numbers is 10, find the other number.
   (a) 15
   (b) 20
   (c) 40
   (d) None of these

4. A 2-digit number exceeds by 19 the sum of the squares of its digits and by 44 the double product of its digits. Find the number
   (a) 72   (b) 62
   (c) 22   (d) 12

5. Which one of the following is a prime number ?
   (a) 161   (b) 221
   (c) 373   (d) 437

Summation 'n' (Σn), Σn^2 and Σn^3

6. Find the sum of first fifty natural numbers.
   (a) 1144   (b) 1275
   (c) 1325   (d) 1075

7. Find the sum of square of first 30 natural numbers.
   (a) 9455   (b) 8372
   (c) 7849   (d) 6973

8. Find the value of $1^2 + 3^2 + 5^2 + 7^2 + \ldots + 19^2$.
   (a) 1335   (b) 1330
   (c) 1332   (d) 1334

9. Find the value of $11^2 + 12^2 + 13^2 + 14^2 + \ldots + 20^2$.
   (a) 2575   (b) 2485
   (c) 2585   (d) 2675

10. Find the value of $1^3 + 3^3 + 5^3 + 7^3 + \ldots + 29^3$.
    (a) 36100   (b) 101025
    (c) 32500   (d) 44700

11. Find the sum of all even numbers up to 100.
    (a) 2295   (b) 2425
    (c) 2495   (d) 2550

12. Find the sum of first twenty five odd number.
    (a) 375   (b) 525
    (c) 475   (d) 625

$$xy - yx = 9|x - y|, \quad xyz - zyx = 99|x - z|,$$
$$xy + yx = 11(x + y)$$

13. If the sum of a 2-digit number and a number formed by reversing it's digit is 99, then what is the sum of the digits of the original number ?
    (a) 9   (b) 81
    (c) 11   (d) 18

14. If the difference of a 2-digit number and a number formed by reversing it's digit is N, which one of the following number will completely divide N ?
    (a) 9   (b) 81
    (c) 11   (d) 18

15. The sum of both digits of a 2-digit number is 7. If the digits of the number are inter changed, the number so formed is greater than the original number by 27 find the original number.
    (a) 29   (b) 25
    (c) 79   (d) 32

ANSWER KEY

1. (c)	28. (c)	55. (a)	82. (b)	109. (c)	136. (d)	163. (c)	190. (b)
2. (d)	29. (c)	56. (d)	83. (c)	110. (c)	137. (a)	164. (a)	191. (c)
3. (b)	30. (b)	57. (c)	84. (c)	111. (b)	138. (a)	165. (c)	192. (d)
4. (a)	31. (d)	58. (b)	85. (a)	112. (a)	139. (b)	166. (b)	193. (b)
5. (c)	32. (c)	59. (b)	86. (a)	113. (a)	140. (b)	167. (b)	194. (d)
6. (b)	33. (b)	60. (b)	87. (a)	114. (c)	141. (c)	168. (b)	195. (d)
7. (a)	34. (b)	61. (a)	88. (a)	115. (d)	142. (a)	169. (c)	196. (c)
8. (b)	35. (d)	62. (b)	89. (a)	116. (d)	143. (d)	170. (a)	197. (d)
9. (b)	36. (c)	63. (c)	90. (a)	117. (b)	144. (c)	171. (c)	198. (a)
10. (b)	37. (c)	64. (c)	91. (d)	118. (a)	145. (b)	172. (b)	199. (a)
11. (d)	38. (c)	65. (d)	92. (b)	119. (a)	146. (a)	173. (b)	200. (b)
12. (d)	39. (a)	66. (a)	93. (b)	120. (d)	147. (c)	174. (d)	201. (a)
13. (a)	40. (c)	67. (b)	94. (b)	121. (a)	148. (c)	175. (a)	202. (a)
14. (a)	41. (d)	68. (b)	95. (c)	122. (b)	149. (d)	176. (c)	203. (a)
15. (b)	42. (a)	69. (b)	96. (a)	123. (d)	150. (c)	177. (d)	204. (c)
16. (a)	43. (b)	70. (b)	97. (b)	124. (a)	151. (d)	178. (c)	205. (b)
17. (d)	44. (c)	71. (c)	98. (c)	125. (c)	152. (c)	179. (b)	206. (a)
18. (d)	45. (a)	72. (a)	99. (a)	126. (b)	153. (c)	180. (b)	207. (c)
19. (c)	46. (d)	73. (a)	100. (b)	127. (a)	154. (a)	181. (d)	208. (d)
20. (a)	47. (c)	74. (c)	101. (d)	128. (d)	155. (a)	182. (d)	209. (c)
21. (b)	48. (d)	75. (b)	102. (c)	129. (c)	156. (c)	183. (a)	210. (b)
22. (c)	49. (b)	76. (c)	103. (d)	130. (b)	157. (c)	184. (b)	211. (c)
23. (b)	50. (d)	77. (c)	104. (c)	131. (a)	158. (b)	185. (a)	212. (c)
24. (c)	51. (b)	78. (b)	105. (d)	132. (a)	159. (d)	186. (c)	213. (c)
25. (c)	52. (c)	79. (c)	106. (c)	133. (a)	160. (b)	187. (d)	
26. (b)	53. (b)	80. (d)	107. (d)	134. (b)	161. (a)	188. (c)	
27. (b)	54. (b)	81. (a)	108. (c)	135. (a)	162. (c)	189. (b)	

Explanation of Selected Questions

1. $2^{73} - 2^{72} - 2^{71} = 2^{71}(2^2 - 2 - 1) = 2^{71}(1)$

2. Take $p = 7, 11, 13$ and check for the options.

3. $x + y = 3(x - y) \rightarrow 2x = 4y$.
   If we take y as 10, we would get the value of x as 20.

4. Going through options we observe that both the condition are satisfied for option (a).

16. Since, $xy - yx = 9 \mid x - y \mid$
    $xyz - zyx = 99 \mid x - z \mid$
    Hence, the result will be multiple of 99, so the remainder is 0.

17. Since, $xyz - zyx = 99 \mid x - z \mid$

20. The unit digit in this case would be '0' because the given expression has 82×85.

21. The units digit would be given by $5 + 6 + 9$ (number ending in 5 and 6 would always end in 5 and 6 irrespective of the power and 3^{54} will give a units digit equivalent to 3^{4n+2} which would give us a unit digit of 3^2 *i.e.,* 9).

22. The respective units digits for the three parts of the expression would be
    $5 + 9 + 2 = 16$. So, required answer is 6.

23. The respective units digits for the six parts of the expression would be
    $1 + 4 + 7 + 6 + 5 + 6 = 29$.
    So, required answer is 9.

24. $5 \times 7 \times 6 = 0$. Option (c) is correct.

27. $\dfrac{157}{5} = 31$

    $\dfrac{31}{5} = 6$

    $\dfrac{6}{5} = 1$

    $31 + 6 + 1 = 38$.

28. Number of 2's in 157!

    $\dfrac{157}{2} + \dfrac{157}{4} + \dfrac{157}{8} + \ldots\ldots + \dfrac{157}{128}$

    $= \left[\dfrac{157}{2}\right] + \left[\dfrac{157}{4}\right] + \left[\dfrac{157}{8}\right] + \ldots\ldots + \left[\dfrac{157}{128}\right]$

    $= 78 + 39 + 19 + 9 + 4 + 2 + 1 = 152$.

    Hence, number of 2^2's would be $\left[\dfrac{152}{2}\right] = 76$.

    Number of 3's in 157! $= 52 + 17 + 5 + 1 = 75$
    The answer would be given by the lower of these values. i.e. 75

29. 77! has 18 zeroes
    42! has 9 zeroes
    So, $18 + 9 = 27$.

30. The number of zeroes would depend on the number of 5's in the value of the factorial.
    100! would end in 24 zeroes
    200! would end in 49 zeroes
    When you add the two numbers (one with 24 zeroes and the other with 49 zeroes at its end), the resultant total would end in 24 zeroes.

31. The given expression has fifteen 2's and seventeen 5's. The number of zeroes would be 15 as the number of 2's is lower in this case.

32. 1! to 4! would have no zeroes while 5! to 9! all the values would have 1 zero. Thus, a total of 5 zeroes till 9!. Going further 10! to 14! would have two zeroes each; similarly we can say,
    Number of zeroes between,
    $15! \times 16! \ldots \times 19! = 3 + 3 + 3 + 3 + 3 = 3 \times 5 = 15$.
    Number of zeroes between,
    $20! \times 21! \ldots \times 24! = 4 \times 5 = 20$.
    Number of zeroes between,
    $25! \times 26! \ldots \times 29! = 6 \times 5 = 30$.
    Number of zeroes between,
    $30! \times 31! \ldots \times 34! = 7 \times 5 = 35$.

Previous Year Solved Questions

Numbers

1. A gardener has 1000 plants. He wants to plant them in such a way that the number of rows and the number of columns remains the same. What is the minimum number of plants that he needs more for this purpose ?
   (a) 14
   (b) 24
   (c) 32
   (d) 34   **[CSAT 2013]**

2. A person is standing on the first step from the bottom of a ladder. If he has to climbs 4 more steps to reach exactly the middle step, how many steps does the ladder have ?
   (a) 8
   (b) 9
   (c) 10
   (d) 11   **[CSAT 2016]**

3. There are some nectar-filled flowers on a tree and some bees are hovering on it. If one bee lands on each, flower, one bee will be left out. If two bees land on each flower, one flower will be left out. The number of flowers and bees respectively are
   (a) 2 and 4
   (b) 3 and 2
   (c) 3 and 4
   (d) 4 and 3

   [CSAT 2016]

4. In aid of charity, every student in a class contributes as many rupees as the number of students in that class. With the additional contribution of Rs. 2 by one student only, the total collection is Rs. 443. Then how many students are there in the class ?
   (a) 12
   (b) 21
   (c) 43
   (d) 45   **[CSAT 2016]**

5. How many numbers are there between 100 and 300 which either begin with or end with 2 ?
   (a) 110
   (b) 111
   (c) 112
   (d) None of the above

   [CSAT 2016]

6. How many numbers are there between 99 and 1000 such that the digit 8 occupies the units place ?
   (a) 64
   (b) 80
   (c) 90
   (d) 104   **[CSAT 2017]**

7. The age of Mr. X last year was the square of a number and it would be the cube of a number next year. What is the least number of years he must wait for his age to become the cube of a number again ?
   (a) 42
   (b) 38
   (c) 25
   (d) 16   **[CSAT 2017]**

8. If X is between -3 and -1, and Y is between -1 and 1, then $X^2 - Y^2$ is in between which of the following ?
   (a) -9 and 1
   (b) -9 and -1
   (c) 0 and 8
   (d) 0 and 9

   [CSAT 2018]

9. X and Y are natural numbers other than 1, and Y is greater than X. Which of the following represents the largest number ?
   (a) XY
   (b) X/Y
   (c) Y/X
   (d) $(X + Y)/XY$

   [CSAT 2018]

10. If $x - y = 8$, then which of the following must be true ?
    1. Both x and y must be positive for any value of x and y.
    2. If x is positive, y must be negative for any value of x and y.
    3. If x is negative, y must be positive for any value of x and y.

    Select the correct answer using the code given below.
    (a) 1 only
    (b) 2 only
    (c) Both 1 and 2
    (d) Neither 1 nor 2

    [CSAT 2018]

101. There are three traffic signals. Each signal changes colour from green to red and then from red to green. The first signal takes 25 seconds, the second signal takes 39 seconds and the third signal takes 60 seconds to change the colour from green to red. The durations for green and red colours are same. At 2:00 pm, they together turn green. At what time will they change to green next, simultaneously ?

(a) 4:00 pm   (b) 4:10 pm

(c) 4:20 pm   (d) 4:30 pm

[CSAT 2023]

Factors
---

102. Let x be a positive integer such that $7x + 96$ is divisible by x. How many values of x are possible ?

(a) 10   (b) 11

(c) 12   (d) Infinitely many

[CSAT 2023]

103. How many natural numbers are there which give a remainder of 31 when 1186 is divided by these natural numbers ?

(a) 6   (b) 7

(c) 8   (d) 9   **[CSAT 2023]**

Greatest and least value
---

104. What is the largest number among the following ?

(a) $\left(\dfrac{1}{2}\right)^{-6}$ (b) $\left(\dfrac{1}{4}\right)^{-3}$

(c) $\left(\dfrac{1}{3}\right)^{-4}$ (d) $\left(\dfrac{1}{6}\right)^{-2}$ **[CSAT 2020]**

105. Which number amongst 2^{40}, 3^{21}, 4^{18} and 8^{12} is the smallest ?

(a) 2^{40} (b) 3^{21}

(c) 4^{18} (d) 8^{12} **[CSAT 2022]**

ANSWER KEY

1. (b)	15. (b)	29. (d)	43. (c)	57. (b)	71. (a)	85. (a)	99. (d)
2. (b)	16. (b)	30. (d)	44. (a)	58. (c)	72. (b)	86. (b)	99. (b)
3. (c)	17. (d)	31. (b)	45. (d)	59. (b)	73. (c)	87. (d)	100. (b)
4. (b)	18. (b)	32. (c)	46. (a)	60. (b)	74. (b)	88. (a)	101. (b)
5. (a)	19. (c)	33. (b)	47. (b)	61. (b)	75. (c)	89. (a)	102. (c)
6. (c)	20. (a)	34. (c)	48. (c)	62. (b)	76. (a)	90. (b)	103. (d)
7. (b)	21. (c)	35. (c)	49. (a)	63. (d)	77. (d)	91. (b)	104. (c)
8. (d)	22. (c)	36. (d)	50. (d)	64. (d)	78. (b)	92. (d)	105. (b)
9. (a)	23. (d)	37. (a)	51. (c)	65. (b)	79. (d)	93. (b)	
10. (d)	24. (c)	38. (d)	52. (d)	66. (b)	80. (a)	94. (b)	
11. (c)	25. (c)	39. (d)	53. (a)	67. (b)	81. (c)	95. (b)	
12. (c)	26. (b)	40. (c)	54. (b)	68. (d)	82. (a)	96. (d)	
13. (d)	27. (b)	41. (d)	55. (c)	69. (c)	83. (c)	97. (b)	
14. (c)	28. (a)	42. (d)	56. (c)	70. (a)	84. (d)	98. (c)	

Hints and Solutions

1. Number of rows = number of column → possible only if a square formed.

   Hence, we used to find a perfect square which is closed to 1000.

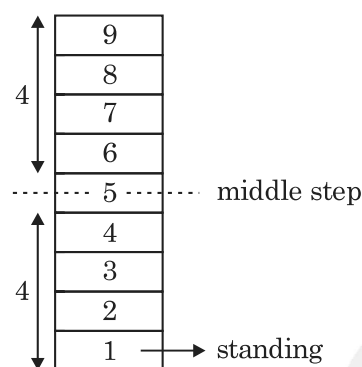 $2 \times 2 \quad 3 \times 3 \quad 4 \times 4$
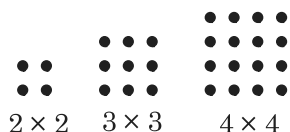
   We know $32^2 = 1024$

   So we can 32 row × 32 column = 1024

   So, gardener need 24 more plants

   $1000 + 24 = 1024 - 1000 = 24$.

2.

   $4 + 4 + 1 = 9$

3. We can go through options:

   (a) 2 flowers 4 bee → ⊙ ⊙ 2 bee left

   (b) 3 flowers 2 bee → ⊙ ⊙ ◯ 1 flower left

   (c) 3 flowers 4 bee → ⊙ ⊙ ⊙ 1 bee left

   ⊙ ⊙ ◯ 1 flower left. If 2 bee on 1 flower.

4. Let x student are there in the class

   Each contribute Rs. x

   So, total contribution = Rs. x^2

   One student contributed addition = Rs. 2

   According to the question,

   So, $x^2 + 2 = 443$, or, $x^2 = 441$

   Hence, $x = 21$.

5. Number ending with 2 → 102, 112, 122, 132, 142, 152, 162, 172, 182, 192 = 10 numbers.

   Number beginning with 2 → 201 to 299 = 100 numbers

   So, total we have = 100 + 10 = 110 numbers either begin or end with 2.

6. From 99 to 199 → 108, 118, 128, 138, 148, 158, 168, 178, 188, 198 → 10 numbers

   Similarly, from 200 to 299 → 10 numbers

   300 to 399 → 10 numbers

   400 to 499 → 10 numbers

   500 to 599 → 10 numbers

   600 to 699 → 10 numbers

   700 to 799 → 10 numbers

   800 to 899 → 10 numbers

   900 to 1000 → 10 numbers

   So, total 10 × 9 = 90.

7.

Last year age	Present age	Next year age
−1	0	1
6	7	8
25	26	27
62	63	64
123	124	125

   His present age = 26

   Last year age = 25 (square)

   Next year age = 27 (cube)

   He has to wait for his age to become 64 years.

   Hence, he has to wait till 64 − 26 = 38 years.

8. $-3 < x < -1$, $-1 < y < 1$

   So, $x^2 = 1$ to 9, $y^2 = 0$ to 1

   $x^2 - y^2$ → maximum, if x^2 is maximum and y^2 minimum.

   So, $9 - 0 = 9$

   $x^2 - y^2$ → minimum, if x^2 is minimum and y^2 maximum.

   So, $1 - 1 = 0$

95. HCF (7.5, 3.25)

$7.5 = \dfrac{75}{10}, \ 3.25 = \dfrac{325}{100}$

$\text{HCF}\left(\dfrac{75}{10}, \dfrac{325}{100}\right) = \dfrac{\text{HCF }(75,\ 325)}{\text{LCM }(10,\ 100)}$

$= \dfrac{25}{100} = 0.25$

96. $\text{HCF}\left(3\dfrac{1}{2}, \ 8\dfrac{3}{4}\right)$

$\text{HCF}\left(\dfrac{7}{2}, \dfrac{35}{4}\right) = \dfrac{\text{HCF }(7,\ 35)}{\text{LCM }(2,\ 4)}$

$\text{HCF} = \dfrac{7}{4} = 1\dfrac{3}{4} \text{ m}$

97. $\text{LCM}(3, 2^2, 5, 2 \times 3) = 2^2 \times 3 \times 5 = 60$

Largest 3-digit number multiple of 60 = 960.

First 4-digit number multiple of 60 = 960 + 60 = 1020

2 is remainder here.

So, number should be 1020 + 2 = 1022.

98. LCM(6, 9, 12, 15, 18)

$6 = 2 \times 3, \ 9 = 3^2, \ 12 = 2^2 \times 3, \ 15 = 3 \times 5,$

$18 = 2 \times 3^2$

$\text{LCM} = 2^2 \times 3^2 \times 5 = 180$

$180 \times 6 = 1080$ smallest 4-digit multiple of 180 but 3 is remainder so 1083.

99. Seeta goes after a gap of 2 days and Geeta goes after a gap of 3 days.

Seeta = 1, 4, 7, 10, **13**, 16, ...

Geeta = 1, 5, 9, **13**, 17, ...

After 1$^{\text{st}}$ January, they will together go for swim on 13$^{\text{th}}$ January.

101. LCM of (25, 39, 60) = 3900

Time required for a signal from green to next green = 2 × 3900 = 7800 seconds

= 2 hours + 10 minutes

= 2 pm + 2 hours and 10 minutes

= 4:10 pm

102. If $7x + 96$ is divisible by x, means x is the factors of 96. Here we need to find total number of factors of 96.

96 can be factorized as = $2^5 \times 3$.

Total factors of 96 = (5 + 1) (1 + 1) = 6 × 2 = 12

Hence, x can here 12 values.

103. $1186 - 31 = 1155$.

Total factors of 1155 which are less than 31 will be the answer of this question i.e. 9.

104. $\left(\dfrac{1}{2}\right)^{-6} = 2^6 = 64$

$\left(\dfrac{1}{4}\right)^{-3} = 4^3 = 2^6 = 64$

$\left(\dfrac{1}{3}\right)^{-4} = 3^4 = 81$ largest number

$\left(\dfrac{1}{6}\right)^{-2} = 6^2 = 36$

105. $2^{40}, 3^{21}, 4^{18}, 8^{12}$

$2^{40}, 3^{21}, (2^2)^{18}, (2^3)^{12}$

$2^{40}, 3^{21}, 2^{36}, 2^{36}$

$2^{40} = \text{not smallest} = 4^{20}$

$3^{21} = \text{smallest} = 3^{31}$

Using options answer cannot (c) and (d) both same and we already eliminated.

Additional Questions from Other Competitive Exams

Numbers

1. n is a natural number. If n^5 is odd, which of the following is *true* ?
   A. n is odd
   B. n^3 is odd
   C. n^4 is even
   (a) A only
   (b) B only
   (c) C only
   (d) A and B only

2. $4^0 + 4^2 + 4^{-2} + 4^{1/2} + 4^{-1/2} =$
   (a) 4^0
   (b) $4^{2\frac{1}{2}} + 4^{-2\frac{1}{2}}$
   (c) $19\frac{9}{16}$
   (d) $22\frac{9}{16}$

3. If $a + b + c + d + e = 10$ (all positive numbers), then the maximum value of $a \times b \times c \times d \times e$ is
   (a) 12
   (b) 32
   (c) 48
   (d) 72

4. $(25 \div 5 + 3 - 2 \times 4) + (16 \times 4 - 3) =$
   (a) 61
   (b) 22
   (c) $\frac{41}{24}$
   (d) 16

5. How many 9-digit positive integers are there, the sum of squares of whose digits is 2 ?
   (a) 8
   (b) 9
   (c) 10
   (d) 11

6. For real numbers x and y, $x^2 + (y - 4)^2 = 0$. Then the value of $x + y$ is
   (a) 0
   (b) 2
   (c) $\sqrt{2}$
   (d) 4

7. The following sum is
   $1 + 1 - 2 + 3 - 4 + 5 - 6\ldots -20 = ?$
   (a) 10
   (b) −10
   (c) −11
   (d) −9

8. Suppose n is a positive integer. Then $(n^2 + n)(2n + 1)$
   (a) may not be divisible by 2
   (b) is always divisible by 2 but may not be divisible by 3
   (c) is always divisible by 3 but may not be divisible by 6
   (d) is always divisible by 6

9. If $A \times B = 24$, $B \times C = 32$, $C \times D = 48$, then $A \times D$
   (a) cannot be found
   (b) is a perfect square
   (c) is a perfect cube
   (d) is odd

10. If N, E and T are distinct positive integers such that $N \times E \times T = 2013$, then which of the following is the maximum possible sum of N, E and T ?
    (a) 39
    (b) 2015
    (c) 675
    (d) 671

11. Which of the following numbers is a perfect square ?
    (a) 1022121
    (b) 2042122
    (c) 3063126
    (d) 4083128

12. Binomial theorem in algebra gives
    $(1 + x)^n = a_0 + a_1x + a_2x^2 + \ldots\ldots + a_nx^n$; where $a_0, a_1, \ldots\ldots, a_n$ are constants depending on n. What is the sum of $a_0 + a_1 + a_2 + \ldots\ldots + a_n$?
    (a) 2^n
    (b) n
    (c) n^2
    (d) $n^2 + n$

13. Which of the following values is same as $2^{2^{2^2}}$?
    (a) 2^6
    (b) 2^8
    (c) 2^{16}
    (d) 2^{222}

14. If
    $$\begin{array}{r} 2\ a \\ \times\ b\ 2 \\ \hline c\ 6 \\ 8\ 4 \\ \hline 8\ d\ 6 \end{array}$$
    Here a, b, c and d are digits. Then $a + b =$
    (a) 4
    (b) 9
    (c) 11
    (d) 16

42. What is the value of
    $1^2 - 2^2 + 3^2 - 4^2 + 5^2 - ... + 17^2 - 18^2 + 19^2$?
    (a) −5 (b) 12
    (c) 95 (d) 190

> $xy - yx = 9\,|\,x - y\,|, \quad xyz - zyx = 99\,|\,x - z\,|,$
> $xy + yx = 11(x + y)$

43. While calculating the average of twenty 2-digit numbers, digits of one of the numbers got interchanged because of which the average reduced by 1.8. The difference in the values of the digits of the number that were interchanged is
    (a) 1 (b) 2
    (c) 3 (d) 4

Cyclicity (unit digit)

44. What is the last digit of 7^{73}?
    (a) 7 (b) 9
    (c) 3 (d) 1

45. Consider the set of numbers
    $\{17^1, 17^2, ..., 17^{300}\}$
    How many of these numbers end with digit 3?
    (a) 60 (b) 75
    (c) 100 (d) 150

46. What is the last digit of $(2017)^{2017}$?
    (a) 1 (b) 3
    (c) 7 (d) 9

Exponents (Number of zeros)

47. How many digits are there in $2^{17} \times 3^2 \times 5^{14} \times 7$?
    (a) 14
    (b) 15
    (c) 16
    (d) 17

48. We define a function $f(N)$ = sum of digits of N, expressed as decimal number,
    e.g., $f(137) = 1 + 3 + 7 = 11$. Evaluate $f(2^7 3^5 5^6)$.
    (a) 10
    (b) 18
    (c) 28
    (d) 11

Divisibility

49. If n is a positive integer, then
    $n(n + 1)(n + 2)(n + 3)(n + 4)(n + 5)(n + 6)$
    is divisible by
    (a) 3 but not 7 (b) 3 and 7
    (c) 7 but not 3 (d) neither 3 nor 7

50. How many non-negative integers less than 10,000 are there such that the sum of the digits of the number is divisible by three?
    (a) 1112 (b) 2213
    (c) 2223 (d) 3334

51. Which is the odd one out based on a divisibility test?
    154, 286, 363, 474, 572, 682
    (a) 474 (b) 572
    (c) 682 (d) 154

52. If $D = ABC + BCA + CAB$, where A, B and C are decimal digits, then D is divisible by
    (a) 37 and 29
    (b) 37 but not 29
    (c) 29 but not 37
    (d) neither 29 nor 37

53. In a 10-digit mobile number $999ABCDEEE$, A, B, C and D are distinct prime numbers. The mobile number is never divisible by
    (a) 3
    (b) 4
    (c) 5
    (d) 8

54. If n is an even number, then the sum of the first n natural numbers is divisible by
    (a) both n and $(n + 1)$
    (b) n but not $(n + 1)$
    (c) $(n + 1)$ but not n
    (d) neither $(n + 1)$ nor n

55. The sum of the first n even numbers is
    (a) divisible by n and not by $(n + 1)$.
    (b) divisible by $(n + 1)$ and not by n.
    (c) divisible by both n and $(n + 1)$.
    (d) neither divisible by n nor by $(n + 1)$.

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01. (d)	10. (c)	19. (d)	28. (d)	37. (c)	46. (c)	55. (c)	64. (b)
02. (c)	11. (a)	20. (b)	29. (d)	38. (b)	47. (d)	56. (a)	65. (b)
03. (b)	12. (a)	21. (b)	30. (b)	39. (c)	48. (b)	57. (a)	66. (d)
04. (a)	13. (c)	22. (b)	31. (d)	40. (c)	49. (b)	58. (d)	
05. (a)	14. (c)	23. (c)	32. (a)	41. (a)	50. (d)	59. (a)	
06. (d)	15. (d)	24. (d)	33. (b)	42. (d)	51. (a)	60. (b)	
07. (d)	16. (a)	25. (b)	34. (c)	43. (d)	52. (b)	61. (a)	
08. (d)	17. (a)	26. (c)	35. (a)	44. (a)	53. (a)	62. (a)	
09. (b)	18. (d)	27. (c)	36. (a)	45. (b)	54. (c)	63. (c)	

Detailed explanation

1. If n^5 is odd, then n is odd and any power of n is odd.

2. $4^0 + 4^2 + 4^{-2} + 4^{1/2} + 4^{-1/2}$

$= 1 + 16 + \dfrac{1}{4^2} + 2 + \dfrac{1}{\sqrt{4}} = 19 + \dfrac{1}{16} + \dfrac{1}{2}$

$= 19 + \dfrac{1+8}{16} = 19 + \dfrac{9}{16} = 19\dfrac{9}{16}$

3. The product $a \times b \times c \times d \times e$ will be maximum when each entity will be equal

   i.e., $a = b = c = d = e$.

   So, $a = b = c = d = e = 2$.

   Hence, the maximum value of the product
   $= a \times b \times c \times d \times e = 2 \times 2 \times 2 \times 2 \times 2 = 2^5 = 32$.

4. The value of given expression is,

   $(25 \div 5 + 3 - 2 \times 4) + (16 \times 4 - 3)$

   $= (5 + 3 - 8) + (64 - 3) = 61$.

5. The possible 9-digit positive numbers are :
   110000000, 101000000, 100100000,
   100010000, 100001000, 100000100,
   100000010, 100000001
   whose sum of squares of digits is 2.

6. $x^2 + (y - 4)^2 = 0$

   If the sum of 2 perfect squares is 0, then both of them separately must be 0.

   (Since, perfect square cannot be negative)

   So, $x^2 = 0$ or, $x = 0$

   And $(y - 4)^2 = 0$, or, $y - 4 = 0$. So, $y = 4$.

   Hence, $x + y = 0 + 4 = 4$.

7. Given sequence is, $1 + 1 - 2 + 3 - 4 + 5 - 6\ldots -20$, can be written as

   $1 + (1 - 2 + 3 - 4 + 5 - 6\ldots + 19 - 20)$

   We can observe inside the bracket every two terms are giving –1, so we have

   $1 + (-1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1) = 1 - 10 = -9$.

8. $(n^2 + n)(2n + 1) = n(n + 1)(2n + 1)$

   If $n = 1$, then $n(n + 1)(2n + 1) = 1 \times 2 \times 3 = 6$

   If $n = 2$, then $n(n + 1)(2n + 1) = 2 \times 3 \times 5 = 6 \times 5$

   If $n = 3$, then $n(n + 1)(2n + 1) = 3 \times 4 \times 7 = 6 \times 14$ and so on.

   Hence, $n(n + 1)(2n + 1)$ is always divisibly by 6.

9. $A \times B = 24$ ...(1)
   $B \times C = 32$ ...(2)
   $C \times D = 48$ ...(3)

   On multiplying equation (1) and (3), we get

   $A \times B \times C \times D = 24 \times 48$ ...(4)

   On dividing equation (4) by equation (2), we get

   $\dfrac{A \times B \times C \times D}{B \times C} = \dfrac{24 \times 48}{32} = 36 = 6^2$

   Hence, $A \times D$ is a perfect square.

10. From prime factorization, we have

    $2013 = 3 \times 671$

    $2013 = 1 \times 3 \times 671$ is the only possible way in which 2013 can be expressed as the product of three distinct positive integers.

    Sum $= 1 + 3 + 671 = 675$.

11. Here, 1022121 is perfect square number which is square of 1021.

12. $(1 + x)^n = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$

    Put the value of $x = 1$, we have

    $2^n = a_0 + a_1 + a_2 + \ldots + a_n$

13. By applying the laws of exponents, we get

    $2^{2^{2^2}} = 2^{[2^{(2^2)}]} = 2^{[2^4]} = 2^{16}$

14.
    $\begin{array}{r} 2\ a \\ \times\ b\ 2 \\ \hline c\ 6 \\ 8\ 4\ \\ \hline 8\ d\ 6 \end{array}$

    The possible values of 'a' are 3 and 8.

    For $a = 3$, we have $23 \times b = 84$

    Or, $b = 84/23$, which is not possible, so 'a' should be 8.

    If $a = 8$, we have $28 \times b = 84$

    So, $b = 3$. Hence, $a + b = 8 + 3 = 11$.

Ratio & Proportion

A ratio is a comparison of two numbers by division, where the first number is divided by the second (obviously non zero) number.

Since ratio is the quotient of two numbers divided in a definite order, so it should be taken care to write each ratio in that particular order.

For example, the ratio of 5 to 3 should be expressed in the following manner.

$\frac{5}{3}$ (fraction from) or $5 : 3$ (colon form), since the ratio is a fraction, so if we multiply or divide both the terms of a ratio with a same number, the ratio does not get affected.

EXAMPLES

1. Continued ratio

Comparison can also be made for more than two quantities.

Let the length of a cuboidal block is 50 cm, breadth is 60 cm, and height is 70 cm, then the ratio of length, breadth and the height is $50 : 60 : 70$, and since all the values are divisible by 10, so after division we get the ratio $5 : 6 : 7$ which is in simplest form.

The first term is known as antecedent and the second term is known as consequent.

For example, In the ratio $5 : 9$, 5 is antecedent and 9 is consequent.

2. Proportion

It is expressed as $a : b :: c : d$, means the way 'a' is related to 'b', the same way 'c' is related to 'd'.

Hence, a proportion is an equation that tells us the two ratios are equal.

Mathematically it can be expressed as $a : b :: c : d$, which means $\frac{a}{b} = \frac{c}{d}$ and by cross multiplication we get $ad = bc$.

Here the outer terms 'a' and 'd' are called the extremes of the proportion and the inner term 'b' and 'c' are called the means.

$a : b :: c : d \Rightarrow a \times d = b \times c$

So, product of extremes = product of means.

3. Fourth proportional

If $a : b :: c : d$, then 'd' is called fourth proportional to a, b, c.

For example, Find the fourth proportional to 4, 9 and 12.

Solution: Let the fourth proportional to 4, 9 and 12 be x.

Then, $4 : 9 : 12 : x \Rightarrow 4 \times x = 9 \times 12$

$x = \frac{9 \times 12}{4} = 27$.

Fourth proportional to 4, 9 and 12 is 27.

Practice Set

1. If $A : B = \dfrac{1}{2} : \dfrac{3}{8}$, $B : C = \dfrac{1}{3} : \dfrac{5}{9}$ and $C : D = \dfrac{5}{6} : \dfrac{3}{4}$, then the ratio $A : B : C : D$ is
   (a) 4 : 6 : 8 : 10
   (b) 6 : 4 : 8 : 10
   (c) 6 : 8 : 9 : 10
   (d) 8 : 6 : 10 : 9

2. The ratio of $4^{3.5} : 2^5$ is same as
   (a) 2 : 1   (b) 4 : 1
   (c) 7 : 5   (d) 7 : 10

3. If $\dfrac{1}{5} : \dfrac{1}{x} = \dfrac{1}{x} : \dfrac{1}{1.25}$, then the value of x is
   (a) 1.5   (b) 2
   (c) 2.5   (d) 3.5

4. If $x : y = 5 : 2$, then $(8x + 9y) : (8x + 2y)$ is
   (a) 22 : 29   (b) 26 : 61
   (c) 29 : 22   (d) 61 : 26

5. If 15% of x = 20% of y, then $x : y$ is
   (a) 3 : 4   (b) 4 : 3
   (c) 17 : 16   (d) 16 : 17

6. The ratio of three numbers is 3 : 4 : 5 and the sum of their squares is 1250. The sum of the numbers is
   (a) 30   (b) 50
   (c) 60   (d) 90

7. Two numbers are respectively 20% and 50% more than a third number. The ratio of the two numbers is
   (a) 2 : 5   (b) 3 : 5
   (c) 4 : 5   (d) 6 : 7

8. If a carton containing a dozen mirrors is dropped, which of the following cannot be the ratio of broken mirrors to unbroken mirrors ?
   (a) 2 : 1   (b) 3 : 1
   (c) 3 : 2   (d) 7 : 5

9. If Rs. 510 be divided A, B, C in such a way that A gets $\dfrac{2}{3}$ of what B gets and B gets $\dfrac{1}{4}$ of what C gets, then their shares are respectively
   (a) Rs. 120, Rs. 240, Rs. 150
   (b) Rs. 60, Rs. 90, Rs. 360
   (c) Rs. 150, Rs. 300, Rs. 60
   (d) None of these

10. Between two railway stations the 1st, 2nd and 3rd class fares are in the ratio of $10 : 8 : 3$ and in a year in a year the ratio of passengers in 1st, 2nd and 3rd class was $3 : 4 : 10$ respectively. If the total sales proceeds of the ticket during a year was Rs. 8050, then find the amount for which the tickets of 2nd class during the reservation ?
    (a) Rs. 2400   (b) Rs. 2600
    (c) Rs. 2800   (d) Rs. 3200

11. Seats for Mathematics, Physics and Biology in a school are in the ratio 5 : 7 : 8. There is a proposal to increase these seats by 40%, 50% and 75% respectively. What will be the ratio of the total seats ?
    (a) 2 : 3 : 4
    (b) 6 : 7 : 8
    (c) 6 : 8 : 9
    (d) None of these

12. A sum of money is to be distributed among A, B, C, D in the proportion of 5 : 2 : 4 : 3. If C gets Rs. 1000 more than D, what is B's share ?
    (a) Rs. 500
    (b) Rs. 1500
    (c) Rs. 2000
    (d) None of these

13. If 40% of a number is equal to two-third of another number, what is the ratio of first number to the second number ?
    (a) 2 : 5   (b) 3 : 7
    (c) 5 : 3   (d) 7 : 3

Chapter 3 — Mixture & Alligation

Mixture and alligation is a very important and easy topic in arithmetic, which is applicable to many problems.

As we all know how to find the average of the given data but when it comes to some complex cases of weighted average, alligation can be a very useful tool to solve those questions.

If two ingredients are mixed together, there must be a cheaper variety and another will be a dearer one and we will get a mixture whose price will be somewhere between cheaper and dearer price depending upon the ratios of ingredients in the mixture.
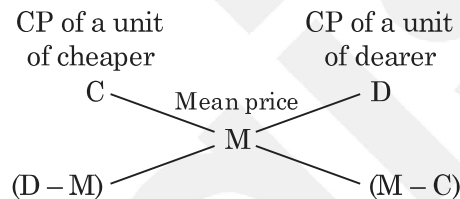
(Cheaper quantity) : (Dearer quantity) = $(d - m) : (m - c)$

EXAMPLES

1. In what ratio must a grocer mix two varieties of sugar costing Rs. 15 per kg and Rs. 20 per kg to get a mixture worth Rs. 16.50 per kg.

Sol: Using alligation :
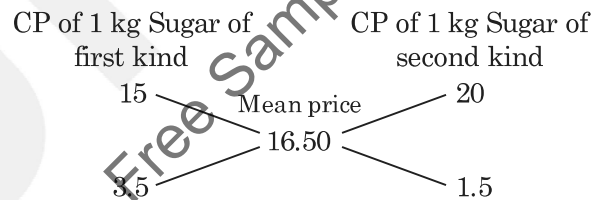
Required ratio = 3.5 : 1.5 = 35 : 15 = 7 : 3.

2. In what ratio rice at Rs. 9.30 per kg must be mixed with rice at Rs. 10.80 per kg as to get the mixture worth Rs. 10 per kg.

Sol: Using alligation :
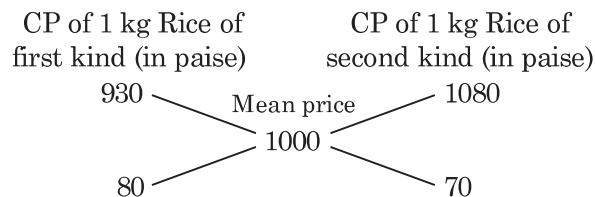
Required ratio = 80 : 70 = 8 : 7.

Practice Set

1. In what ratio must a grocer mix two varieties of pulses costing Rs. 15 and Rs. 20 per kg respectively so as to get a mixture worth Rs. 16.50 per kg ?
    (a) 3 : 7   (b) 5 : 7
    (c) 7 : 3   (d) 7 : 5

2. Find the ratio in which rice at Rs. 7.20 a kg be mixed with rice at Rs. 5.70 a kg to produce a mixture worth Rs. 6.30 a kg.
    (a) 1 : 3   (b) 2 : 3
    (c) 3 : 4   (d) 4 : 5

3. In what ratio must tea at Rs. 62 per kg be mixed with tea at Rs. 72 per kg so that the mixture must be worth Rs. 64.50 per kg ?
    (a) 3 : 1   (b) 3 : 2
    (c) 4 : 3   (d) 5 : 3

4. In what ratio must water be mixed with milk costing Rs. 12 per litre to obtain a mixture worth of Rs. 8 per litre ?
    (a) 1 : 2   (b) 2 : 1
    (c) 2 : 3   (d) 3 : 2

5. The average of marks obtained by 120 candidates in a certain examination is 35. If the average marks obtained by passed candidates are 39 and those of the failed candidates are 15, what is the number of candidates who passed the examination ?
    (a) 100
    (b) 120
    (c) 150
    (d) 140

6. The average salary of all the workers in a workshop is Rs. 8000. The average salary of 7 technicians is Rs. 12000 and the average salary of the rest is Rs. 6000. The total number of workers in the workshop is
    (a) 20   (b) 21
    (c) 23   (d) 22

7. In a family of 8 adults and some minors, the average consumption of rice per head per month is 10.8 kg, while the average consumption for adults is 15 kg per head and for minors it is 6 kg per head. The number of minors in the family is
    (a) 8   (b) 6
    (c) 7   (d) 9

8. The average daily wages of some workers of a factory is Rs. 92. There are 300 male and 200 female workers working in the factory. Each female worker receives Rs. 20 less than a male worker. The daily wages of a male worker is
    (a) Rs. 80   (b) Rs. 96
    (c) Rs. 100   (d) Rs. 120

9. The average of marks scored by the students of a class is 68. The average of marks of the girls in the class is 80 and that of boys is 60. What is the percentage of boys in the class?
    (a) 40   (b) 60
    (c) 6   (d) 70

10. The average monthly salary of the workers in a workshop is Rs. 8500. If the average monthly salary of 7 technicians is Rs. 10000 and average monthly salary of the rest is Rs. 7800 the total number of workers in the workshop is
    (a) 18   (b) 20
    (c) 22   (d) 24

11. The average mathematics marks of two sections *A* and *B* of class IX in the annual examination is 74. The average marks of section *A* is 77.5 and that of section *B* is 70. The ratio of the number of students of section *A* and *B* is
    (a) 7 : 8
    (b) 7 : 5
    (c) 8 : 7
    (d) 8 : 5

24. A vessel contains 24 litres of milk. 4 litres are withdrawn and replaced by water. The process is repeated second time. Find the ratio of milk to water in the resulting mixture ?

    (a) 25 : 36   (b) 36 : 11

    (c) 11 : 25   (d) 25 : 11

25. In a Zoo, there are some Dears and Ducks. If the head are counted, they are 180, while the legs are 448. What will be the number of Dears in the Zoo ?

    (a) 136   (b) 68

    (c) 44   (d) 22

ANSWER KEY

1. (c)	5. (a)	9. (b)	13. (d)	17. (c)	20. (a)	22. (c)	24. (d)
2. (b)	6. (b)	10. (c)	14. (b)	18. (c)	21. (a)	23. (d)	25. (c)
3. (a)	7. (c)	11. (c)	15. (a)	19. (c)			
4. (a)	8. (c)	12. (d)	16. (d)				

Previous Year Questions

1. Two glasses of equal volume are respectively half and three-fourths filled with milk. They are then filled to the brim by adding water. Their contents are then poured into another vessel. What will be the ratio of milk to water in this vessel ?

    (a) 1 : 3   (b) 2 : 3

    (c) 3 : 2   (d) 5 : 3   **[CSAT 2012]**

2. Two equal glasses of same type are respectively $\frac{1}{3}$ and $\frac{1}{4}$ full of milk. They are then filled up with water and the contents are mixed in a pot. What is the ratio of milk and water in the pot ?

    (a) 7 : 17   (b) 1 : 3

    (c) 9 : 21   (d) 11 : 23

    [CSAT 2015]

3. 30 g of sugar was mixed in 180 ml water in a vessel A, 40 g of sugar was mixed in 280 ml of water in vessel B and 20 g of sugar was mixed in 100 ml of water in vessel C. The solution in vessel B is

    (a) sweeter than that in C.

    (b) sweeter than that in A.

    (c) as sweet as that in C.

    (d) less sweet than that in C.   **[CSAT 2016]**

4. The monthly average salary paid to all the employees of a company was Rs. 5000. The monthly average salary paid to male and female employees was Rs. 5200 and Rs. 4200 respectively. Then the percentage of males employed in the company is

    (a) 75%   (b) 80%

    (c) 85%   (d) 90%

    [CSAT 2016]

5. There is a milk sample with 50% water in it. If $\frac{1}{3}$ rd of this milk is added to equal amount of pure milk, then water in the new mixture will fall down to

    (a) 25%   (b) 30%

    (c) 35%   (d) 40%   **[CSAT 2017]**

6. Consider the following data:

	Average marks in English	Average marks in Hindi
Girls	9	8
Boys	8	7
Overall average marks	8.8	x

    What is the value of x in the above table ?

    (a) 7.8   (b) 7.6

    (c) 7.4   (d) 7.2   **[CSAT 2020]**

Problems based on Ages

Concept :

The difference between the ages of two persons does not change with time while the ratio changes in a regular interval.

For example :

Year	Age of 'A'	Age of 'B'	Ratio
2000	16 years	20 years	4 : 5
2004	20 years	24 years	5 : 6
2008	24 years	28 years	6 : 7
2012	28 years	32 years	7 : 8
2016	32 years	36 years	8 : 9
2020	36 years	40 years	9 : 10

In the above table we can easily observe in the interval of 4 years, the ratio of ages is changing regularly. This regular difference of years is known as interval size. To find the age of a person in any given years, we just multiply the interval size with the ratio term corresponding to the person.

Let we need to find age of B in 2008, for that we just multiply interval size i.e. 4 with the ratio term corresponding to B in 2008 *i.e.* 7.

So, the age of B in 2008 will be $7 \times 4 = 28$ years.

1. The present ages of A and B are in the ratio 4 : 5. 8 years hence the ratio will become 5 : 6, find out the present age of A ?

Sol:

	A		B
Present ratio	4	:	5
8 years hence	5	:	6

In 8 years, ratio term of A is changing from 4 to 5 and that of B is changing from 5 to 6. So the interval size is 8 years.

Hence, present age of A = $4 \times 8 = 32$ years and present age of B = $5 \times 8 = 40$ years.

2. The present ages of A and B are in the ratio 7 : 8. 5 years hence the ratio will become 8 : 9, find out the present age of B ?

Sol:

	A		B
Present ratio	7	:	8
5 years hence	8	:	9

In 5 years, ratio term of A is changing from 7 to 8 and that of B is changing from 8 to 9.

So, the interval size is 5 years.

Hence, present age of A = $5 \times 7 = 35$ years and present age of B = $5 \times 8 = 40$ years.

27. 3 years before, the average age of a family of 5 members was 17 years. Due to birth of new child, the average age is same today. The present age of child is

   (a) 1 year   (b) 3 years

   (c) $2\frac{1}{2}$ years   (d) 2 years

28. The age of a man after 15 years is 4 times the age of that man 15 years before. His present age is

   (a) 10 years

   (b) 15 years

   (c) 20 years

   (d) 25 years

29. Ravi's brother is 3 years elder to him. His father was 28 year of age when his sister was born, while his mother was 26 years of age when he was born. If his sister was 4 years of age when his brother was born, the ages of Ravi's father and mother respectively when his brother was born were

   (a) 32 years and 23 years

   (b) 32 years and 29 years

   (c) 35 years and 29 years

   (d) 35 years and 33 years

30. Raman's present age is three times his daughter's and nine-thirteenth of his mother's present age. The sum of the present ages of all three of them is 125 year. What is the difference between the present ages of Raman's daughter and Raman's mother ?

   (a) 45 years

   (b) 50 years

   (c) None of these

   (d) Cannot be determine

31. 16 years ago, Shiwani's grandfather was 8 times as old as her. He would be 3 times of her age 8 years from now. What was the ratio of Shiwani's age to that of her grandfather's age 6 years ago ?

   (a) 1 : 5   (b) 4 : 37

   (c) 5 : 36   (d) 7 : 31

32. Raman's father was 36 years of age when he was born while his mother was 32 years of age when his brother 3 years younger to him was born. What will be the average age of his parents when his brother will be 7 years old ?

   (a) 38 years   (b) 42.5 years

   (c) 34 years   (d) 39.5 years

33. Seema's present age is four times her son's present age and four-seventh of her father's present age. The average of the present ages of all three of them is 32 years. What is the difference between the Seema's son's present age and Seema's father's present age ?

   (a) 54 years   (b) 48 years

   (c) 36 years   (d) 42 years

34. Lokesh's father was 36 years of age when he was born while his mother was 34 years of age when his sister 3 years younger to him was born. What is the sum of ages of his parents when his sister will be 9 years old?

   (a) 77 years   (b) 91 years

   (c) 73 years   (d) 87 years

35. The incomes of A and B are in the ratio 3 : 2 and their expenditures are in the ratio 5 : 3, if each saves Rs. 2500, then find the expenditure of B ?

   (a) Rs. 5000   (b) Rs. 4000

   (c) Rs. 4500   (d) Rs. 7500

ANSWER KEY

1. (c)	6. (b)	11. (c)	16. (b)	21. (c)	26. (d)	31. (d)	34. (b)
2. (b)	7. (c)	12. (b)	17. (a)	22. (a)	27. (d)	32. (b)	35. (d)
3. (b)	8. (c)	13. (d)	18. (b)	23. (c)	28. (d)	33. (b)	
4. (c)	9. (b)	14. (d)	19. (c)	24. (a)	29. (a)		
5. (a)	10. (b)	15. (b)	20. (b)	25. (d)	30. (b)		

Chapter	**5**	**Average**

Average is an equal distribution of total value among all the members of the group or an average of a data set is the central value. Mathematically it is defined as the ratio between sum of data and the number of data.

EXAMPLES

1. Find the average of first ten natural numbers ?

Sol: The average of first ten natural numbers $= \dfrac{1+2+3+......+10}{10} = \dfrac{55}{10} = 5.5.$

2. Find the average of first ten consecutive odd natural numbers ?

Sol: The average of first ten consecutive odd natural numbers $= \dfrac{1+3+5+......+19}{10} = 10.$

3. Find the average of first six consecutive even natural numbers ?

Sol: $\dfrac{2+4+6+8+10+12}{6} = 7.$

4. The average of seven consecutive odd natural numbers is 79. Find the largest among them ?

Sol: The average of seven consecutive odd natural number will be the exactly middle number *i.e.*, 4th term.

These numbers will be 73, 75, 77, 79, 81, 83, 85.

Hence, the largest among those numbers is 85.

5. The average of eight consecutive odd natural numbers is 50. Find the least among them ?

Sol: These numbers will be 43, 45, 47, 49, 50, 51, 53, 55, 57.

Hence, the least among those numbers is 57.

6. Find the average of 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, 6, 7, 7, 7, 7, 7, 7, 7.

Sol: The average will be $\dfrac{1\times1+2\times2+3\times3+4\times4+5\times5+6\times6+7\times7}{1+2+3+4+5+6+7} = \dfrac{\Sigma 7^2}{\Sigma 7} = \dfrac{7\times8\times15}{6\times\dfrac{7\times8}{2}} = 5.$

Contribution concept :

When any quantity joins a group, they bring in surplus or deficit. This surplus/deficit is equally distributed among all the participants including the new entrants.

Practice Set

1. The arithmetic mean (average) of the first 10 whole numbers is
   (a) 5   (b) 4
   (c) 5.5   (d) 4.5

2. The average of the first 100 positive integers is
   (a) 100   (b) 51
   (c) 50.5   (d) 49.5

3. The average of seven consecutive positive integers is 26. The smallest of these integers is
   (a) 21   (b) 23
   (c) 25   (d) 26

4. The average of 5 consecutive natural numbers is m. If the next three natural numbers are also included, how much more than m will the average of these 8 numbers be ?
   (a) 2   (b) 1
   (c) 1.4   (d) 1.5

5. The marks of a student in English, Mathematics, Physics and Chemistry are respectively 59, 83, 75 and 43. Find his average marks
   (a) 66   (b) 63
   (c) 64   (d) 65

6. There are 21 classes in a college. The total number of students in the college is 840. Find the average number of students in each class
   (a) 39   (b) 40
   (c) 41   (d) 44

7. The sum of seven numbers is 235. The average of first three numbers is 23 and the average of last three numbers is 42. Find the fourth number
   (a) 39
   (b) 41
   (c) 40
   (d) 44

8. A man purchased 5 cows at Rs. 1500 each. 6 cows at Rs. 2000 each and 9 cows at Rs. 2500 each. Find the average cost of cows
   (a) Rs. 2200   (b) Rs. 2300
   (c) Rs. 2100   (d) Rs. 2400

9. 30 horses were purchased for Rs. 12000. The average cost of 12 horses out of them is Rs. 250. Find the average cost of the remaining horses
   (a) Rs. 500
   (b) Rs. 600
   (c) Rs. 650
   (d) Rs. 550

10. Total weekly emoluments of the workers of a factory is Rs. 1534. Average weekly emolument of a worker is Rs. 118. The number of workers in the factory is
    (a) 16   (b) 14
    (c) 13   (d) 12

11. A student was asked to find the arithmetic mean of the following 12 numbers 3, 11, 7, 9, 15, 13, 8, 19, 17, 21, 14 and x. He found the mean to be 12. The value of x will be
    (a) 3   (b) 7
    (c) 17   (d) 31

12. The average income of 40 persons is Rs. 4200 and that of another 35 persons is Rs. 4000. The average income of the whole group is
    (a) Rs. 4100   (b) Rs. $4106\frac{1}{3}$
    (c) Rs. $4106\frac{2}{3}$ (d) Rs. $4108\frac{1}{3}$

13. The average of 7 consecutive numbers is 20. The largest of these numbers is
    (a) 24
    (b) 23
    (c) 22
    (d) 20

40. Mean of 10 numbers is 30. Later on it was observed that numbers 15, 23 are wrongly taken as 51, 32. The correct mean is

    (a) 25.5   (b) 32

    (c) 30   (d) 34.5

41. The average of five numbers is 140. If one number is excluded, the average of the remaining four numbers is 130. The excluded number is

    (a) 135   (b) 134

    (c) 180   (d) 150

42. The average of runs scored by a player in 10 innings is 50. How many runs should he score in the 11th innings so that his average is increased by 2 runs ?

    (a) 80 runs   (b) 72 runs

    (c) 60 runs   (d) 54 runs

43. The mean of 50 observations was 36. It was found later that an observation 48 was wrongly taken as 23. The corrected (new) mean is

    (a) 35.2

    (b) 36.1

    (c) 36.5

    (d) 39.1

44. In an examination, the average of marks was found to be 50. For deducting marks for computational errors, the marks of 100 candidates had to be changed from 90 to 60 each and so the average of marks came down to 45. The total number of candidates, who appeared at the examination, was

    (a) 600   (b) 300

    (c) 200   (d) 150

45. While calculating the average of twenty 2-digit numbers, digits of one of the numbers got interchanged because of which the average reduced by 1.8. The difference in the values of the digits of the number that were interchanged is

    (a) 1   (b) 2

    (c) 3   (d) 4

46. While calculating the average of twenty 2-digit numbers, digits of one of the numbers got interchanged because of which the average reduced by 1.8. The difference in the values of the digits of the number that were interchanged is

    (a) 1   (b) 2

    (c) 3   (d) 4

ANSWER KEY

1. (d)	7. (c)	13. (b)	19. (c)	25. (b)	31. (c)	37. (c)	43. (c)
2. (c)	8. (c)	14. (d)	20. (a)	26. (c)	32. (c)	38. (b)	44. (a)
3. (b)	9. (a)	15. (d)	21. (d)	27. (d)	33. (c)	39. (c)	45. (d)
4. (d)	10. (c)	16. (a)	22. (c)	28. (d)	34. (b)	40. (a)	46. (d)
5. (d)	11. (b)	17. (c)	23. (b)	29. (d)	35. (d)	41. (c)	
6. (b)	12. (c)	18. (c)	24. (b)	30. (d)	36. (b)	42. (b)	

Previous Year Questions

1. A student on her first 3 tests received an average score of N points. If she exceeds her previous average score by 20 points on her fourth test, then what is the average score for the first 4 tests?
   (a) $N + 20$ (b) $N + 10$
   (c) $N + 4$ (d) $N + 5$ **[CSAT 2011]**

2. The sum of the ages of 5 members comprising a family, 3 years ago was 80 years. The average age of the family today is the same as it was 3 years ago, because of an addition of a baby during the intervening period. How old is the baby?
   (a) 6 months
   (b) 1 year
   (c) 2 years
   (d) 2 years and 6 months   **[CSAT 2016]**

3. The average monthly income of a person in a certain family of 5 is Rs. 10,000. What will be the average monthly income of a person in the same family if the income of one person increased by Rs. 1,20,000 per year?
   (a) Rs. 12,000   (b) Rs. 16,000
   (c) Rs. 20,000   (d) Rs. 34,000
   [CSAT 2016]

4. Suppose the average weight of 9 persons is 50 kg. The average weight of the first 5 persons is 45 kg, whereas the average weight of the last 5 persons is 55 kg. Then the weight of the 5th person will be
   (a) 45 kg   (b) 47.5 kg
   (c) 50 kg   (d) 52.5 kg
   [CSAT 2017]

5. There are thirteen 2-digit consecutive odd numbers. If 39 is the mean of the first five such numbers, then what is the mean of all the thirteen numbers?
   (a) 47   (b) 49
   (c) 51   (d) 45   **[CSAT 2017]**

6. The average rainfall in a city for the first four days was recorded to be 0.40 inch. The rainfall on the last two days was in the ratio of 4 : 3. The average of six days was 0.50 inch. What was the rainfall on the fifth day?
   (a) 0.60 inch   (b) 0.70 inch
   (c) 0.80 inch   (d) 0.90 inch
   [CSAT 2017]

7. The average marks of 100 students are given to be 40. It was found later that marks of one student were 53 which were misread as 83. The corrected mean marks are
   (a) 39   (b) 39.7
   (c) 40   (d) 40.3 **[CSAT 2019]**

8. A family has two children along with their parents. The average of the weights of the children and their mother is 50 kg. The average of the weights of the children and their father is 52 kg. If the weight of the father is 60 kg, then what is the weight of the mother?
   (a) 48 kg   (b) 50 kg
   (c) 52 kg   (d) 54 kg **[CSAT 2019]**

9. The average age of a teacher and three students is 20 years. If all the three students are of same age and the difference between the age of the teacher and each student is 20 years, then what is the age of the teacher?
   (a) 25 years   (b) 30 years
   (c) 35 years   (d) 45 years
   [CSAT 2020]

10. In a class, there are three groups A, B and C. If one student from group A and two students from group B are shifted to group C, then what happens to the average weight of the students of the class?
    (a) It increases.
    (b) It decreases.
    (c) It remains the same.
    (d) No conclusion can be drawn due to insufficient data.
    [CSAT 2020]

6.1   Fraction values table

Fraction values	Percentage values	Fraction values	Percentage values
$\dfrac{1}{1}$	100%	$\dfrac{1}{11}$	$9\dfrac{1}{11}\%$
$\dfrac{1}{2}$	50%	$\dfrac{1}{12}$	$8\dfrac{1}{3}\%$
$\dfrac{1}{3}$	$33\dfrac{1}{3}\%$	$\dfrac{1}{13}$	$7\dfrac{9}{13}\%$
$\dfrac{1}{4}$	25%	$\dfrac{1}{14}$	$7\dfrac{1}{7}\%$
$\dfrac{1}{5}$	20%	$\dfrac{1}{15}$	$6\dfrac{2}{3}\%$
$\dfrac{1}{6}$	$16\dfrac{2}{3}\%$	$\dfrac{1}{16}$	$6\dfrac{1}{4}\%$
$\dfrac{1}{7}$	$14\dfrac{2}{7}\%$	$\dfrac{1}{17}$	$5\dfrac{15}{17}\%$
$\dfrac{1}{8}$	$12\dfrac{1}{2}\%$	$\dfrac{1}{18}$	$5\dfrac{5}{9}\%$
$\dfrac{1}{9}$	$11\dfrac{1}{9}\%$	$\dfrac{1}{19}$	$5\dfrac{5}{19}\%$
$\dfrac{1}{10}$	10%	$\dfrac{1}{20}$	5%

6.2   Application of fraction values

1.   If $16\dfrac{2}{3}\%$ of a number is added to the number itself, it becomes 1470. Find out the original number ?

Sol: We know that, $16\dfrac{2}{3}\% = \dfrac{1}{6}$

If we add $\dfrac{1}{6}$ of the number to the number itself, it becomes $\dfrac{7}{6}$ of itself.

6.7 Difference between Compound interest and Simple interest

1. Find out of the difference between SI and CI at the rate of 10% per annum for 2 years on Rs. 5000.

Sol: Difference between CI and SI

For 2 years $= \dfrac{r^2}{100}\%$ of Principal

r = rate of interest, P = Principal

Difference between SI and CI $= \dfrac{10^2}{100}\%$ of 5000 = 1% of 5000 = 50.

2. The difference between the compound interest and simple interest on a certain sum at 10% per annum for 2 years is Rs. 631. Find the sum.

Sol: Difference between SI and CI $= \dfrac{10^2}{100}$ of P = 1% of P

1% of P = 631
100% of P = 631 × 100 = 63100.

3. The difference between the compound interest and the simple interest accrued on an amount of Rs. 18,000 in 2 years was Rs. 405. What was the rate of interest per annum?

Sol: $\dfrac{405}{18000} \times 100 = 2.25\%$

$\dfrac{r^2}{100} = 2.25\%$

$r^2 = 225\%$

Hence, r = 15% per annum.

4. Find out the difference between CI and SI on Rs. 40 for 3 years at the rate of 15% per annum.

Sol: r = 15%, t = 3 years and R = Rs. 40

$$40$$
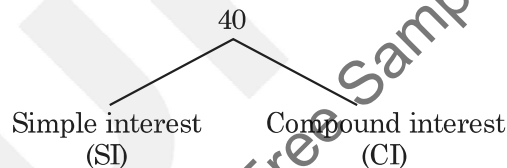
	Simple interest (SI)	Compound interest (CI)
For 1st year :	15% of 40 = 6	6 = 15% of 40
For 2nd year :	15% of 40 = 6	6 + 0.9 = 15% of 40 + 15% of 6
For 3rd year :	15% of 40 = 6	6 + 0.9 + 0.9 + 0.135 = 15% of 40 + 15% of 6 + 15% of 6 + 15% of 0.9

SI = 6 + 6 + 6 = 18

CI = 6 + 6 + 6 + 0.9 + 0.9 + 0.9 + 0.135 = 18 + 2.835

Difference between CI and SI = CI – SI = 18 + 2.835 – 18 = 2.835.

5. Find out the principal and the difference between SI and CI, if the compound interest for third years is Rs. 1200 at the rate of $11\dfrac{1}{9}\%$.

Sol: $r = 11\dfrac{1}{9}\% = \dfrac{1}{9} = \dfrac{1}{a}$

6.8 Compounded half yearly and quarterly

1. Find the compound interest on Rs. 16,000 at 20% per annum, for 9 months, compounded quarterly.

Sol: Here compounding is being done quarterly.

So, rate will be taken as $\frac{20}{4} = 5\%$ per annum and time will be taken as 3 years hence we can say.

20% per annum compounded quarterly for 9 months = 5% per annum compounded annually for 3 years.

$r = 5\% = \frac{1}{20}$ and t = 3 years

Assumed Principal = 20^3 = 8000, Amount = 21^3 = 9261

CI = 9261 – 8000 = 1261 units

According to the question,

8000 units = 16000, then 1 unit = 2

So, 1261 units = 1261 × 2 = Rs. 2522.

2. Find out the compound interest on Rs. 8000 at 20% per annum for 1.5 years, if the interest is being compounded half yearly.

Sol: Here compounding is being done half yearly.

So, rate will be taken as $\frac{20}{2} = 10\%$ per annum and time will be taken as $1.5 \times 2 = 3$ years hence we can say.

20% per annum compounded half yearly for 1.5 years = 10% per annum compounded annually for 3 years.

$r = 10\% = \frac{1}{10}$ and t = 3 years

Final principal = 10^3 = 1000 units and final amount = 11^3 = 1331 units

CI = 331 units

According to the question,

1000 units = 8000, then 1 unit = 8

So, 331 units = 331 × 8 = Rs. 2648.

3. Find out the difference between CI and SI on Rs. 8000 at 20% per annum for 2 years, if the interest is being compounded half yearly.

Sol: A_1 = 10% of 8000 = 800

A_2 = 10% of 800 = 80

A_3 = 10% of 80 = 8

A_4 = 10% of 8 = 0.8

Difference between CI and SI = $6A_2 + 4A_3 + A_4$ = 6(80) + 4(8) + 0.8 = 480 + 32 + 0.8 = 512.8.

Price of Rice = x = Rs. 5 per kg and price of wheat = $5x$ = Rs. 25 per kg

Let the new consumption of rice = y kg

So, 5y + 30 × 9 = 350

y = 16 kg

Old consumption = 25 kg

New consumption = 16 kg

Reduced consumption = 25 – 16 = 9 kg

Reduced percentage consumption $= \dfrac{9}{25} \times 100 = 36\%$.

6.11   Unitary method

1. Neelam spends 20% of her salary as house rent, 70% of remaining as other house hold expenses. If now she is left with Rs. 4800, find out the total salary ?

Sol: Let her salary = 100%

According to the question,


```
                    100%
                   /    \
          Rent 20%       80%
                        /    \
   Miscellaneous 56%      24% = Rs. 4800
                              Savings
```
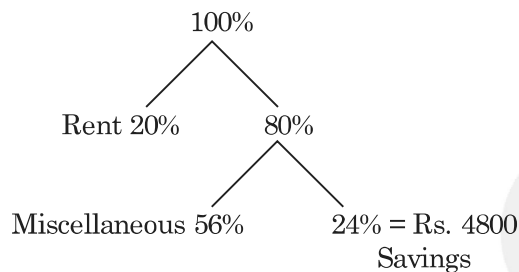
24% of her salary = Rs. 4800

So, 100% of her salary $= \dfrac{4800}{24} \times 100 = $ Rs. 20000.

2. A man gives 30% of his wealth to his wife. 40% of remaining to his son and rest he has equally divided among his three daughters. If each daughter gets Rs. 70000, then find out the share of his wife.

Sol: Let the total wealth = 100%

According to the question,

```
            100%
           /    \
   Wife 30%      70%
                /    \
        Son 28%       42%
                     / | \
                 14% 14% 14%
                 D1  D2  D3
```
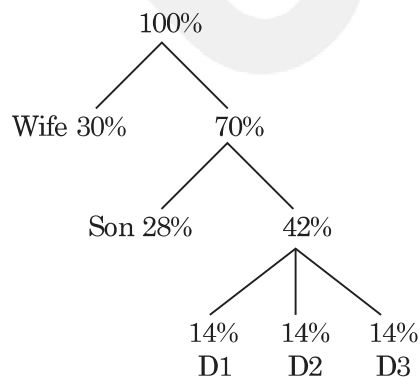
Since, each daughter gets 14% of his wealth and according to the question each daughter gets Rs. 70000

6.12 Complementary thinking

1. Fresh cucumber contains 99% water by weight. Hari bought 100 kg of such cucumber and stored it for 5 days. Now every cucumber contains 98% water by weight. Find the current weight of cucumber.

Sol:

	Water	Pulp
Fresh stage	99%	1%
Dry stage	98%	2%

Weight of fresh stage = 100 kg

Pulp = 1% of 100 kg = 1 kg

This pulp will remain same in dry stage also

In dry stage the concentration of pulp = 2%

For dry stage, 2% = 1 kg

$$100\% = \frac{1}{2} \times 100 = 50 \text{ kg}$$

Hence, reduced weight = 50 kg.

2. A large watermelon of 20 kg contains 96% water by weight. It is stored for 2 days and now it contains 95% water by weight. Find out the reduced weight of the watermelon.

Sol:

	Water	Pulp
Fresh stage	96%	4%
Dry stage	95%	5%

Weight of fresh stage = 20 kg

Pulp = 4% of 20 kg = 0.8 kg

This pulp will remain same in dry stage also, in dry stage the concentration of pulp = 5%

For dry stage, 5% = 0.8 kg

$$100\% = \frac{0.8}{5} \times 100 = 16 \text{ kg}$$

Hence, reduced weight = 16 kg.

3. Fresh grapes contain 90% water by weight and its raisins contain 20% of water by weight. How much fresh fruit is required to convert them into 4 kg of dry fruits?

Sol:

	Water	Pulp
Fresh stage	90%	10%
Dry stage	20%	80%

Weight of dry stage = 4 kg

Pulp = 80% of 4 kg = 3.2 kg

This pulp will remain same in fresh stage also, in fresh stage the concentration of pulp = 10%

For fresh stage, 10% = 3.2 kg

$$100\% = \frac{3.2}{10} \times 100 = 32 \text{ kg}.$$

Hence, reduced weight = 32 kg.

6.15 Miscellaneous

1. The number of boys and girls who appeared at an examination were in the ratio 16 : 9 and the number of boys and girls passing the examination were in the ratio 4 : 3. If 75% of the girls passed the examination, find the percentage of boys who passed the examination and the percentage of all the candidates passing the examination.

Sol: Let number of boys appeared = 1600

Let number of girls appeared = 900

So, total students appeared = 1600 + 900 = 2500

Since 75% girls are passed

So, number of passed girls = 75% of 900 = 675,

The ratio of passed boys : passed girls = 4 : 3

Let boys passed = x

So, $\dfrac{4}{3} = \dfrac{x}{675}$

$x = 900$

The percentage of boys who passed $= \dfrac{900}{1600} \times 100 = 56.25\%$

Total students passed = 900 + 675 = 1575

Percentage of total passed students $= \dfrac{1575}{2500} \times 100 = 63\%$.

2. In an examination paper of 5 question, 5% of the candidates answered all of them and 5% none. Of the rest, 25% answered only 1 question and 20% answered 4. If $24\dfrac{1}{2}\%$ of the entire number answered only 2 questions and 200 candidates answered only 3 questions, how many competed ?

Sol: 0 = 5%

1 = 22.5%

2 = 24.5%

3 = x

4 = 18%

5 = 5%

So, x = 100% – (5% + 22.5% + 24.5% + 18% + 5%) = 25%

According to the question,

25% = 200

$100\% = \dfrac{200}{25} \times 100 = 800$.

53. In a direct election between two candidates, 20% of the voters did not cast their votes and 120 votes were declared to be invalid. If the winner got 41% of the total votes and won by 200 votes. Find the valid votes

    (a) 3080   (b) 3200
    (c) 4000   (d) 3380

Passing marks

54. In an examination, a student who gets 20% of the maximum marks fails by 5 marks. Another student who scores 30% of the maximum marks gets 20 marks more than the pass marks. The necessary percentage required for passing is

    (a) 32%   (b) 23%
    (c) 22%   (d) 20%

55. A scored 30% marks and failed by 15 marks. B scored 40% marks and obtained 35 marks more than those required to pass. The pass percentage is

    (a) 33%   (b) 38%
    (c) 43%   (d) 46%

ANSWER KEY

1. (a)	8. (a)	15. (d)	22. (a)	29. (c)	36. (a)	43. (a)	50. (c)
2. (c)	9. (c)	16. (c)	23. (c)	30. (b)	37. (b)	44. (c)	51. (d)
3. (b)	10. (c)	17. (a)	24. (a)	31. (c)	38. (b)	45. (a)	52. (b)
4. (d)	11. (b)	18. (a)	25. (b)	32. (c)	39. (d)	46. (c)	53. (a)
5. (d)	12. (b)	19. (c)	26. (b)	33. (d)	40. (b)	47. (c)	54. (c)
6. (a)	13. (d)	20. (c)	27. (b)	34. (c)	41. (a)	48. (c)	55. (a)
7. (b)	14. (c)	21. (c)	28. (b)	35. (b)	42. (b)	49. (c)	

Previous Year Questions

1. In a group of persons, 70% of the persons are male and 30% of the persons are married. If two-sevenths of the males are married, what fraction of the females is single ?

   (a) $\dfrac{2}{7}$

   (b) $\dfrac{1}{3}$

   (c) $\dfrac{3}{7}$

   (d) $\dfrac{2}{3}$ **[CSAT 2011]**

2. The tank-full petrol in Arun's motor-cycle lasts for 10 days. If he starts using 25% more everyday, how many days will the tank-full petrol last ?

   (a) 5

   (b) 6

   (c) 7

   (d) 8 **[CSAT 2013]**

3. A and B decide to travel from place X to place Y by bus. A has Rs. 10 with him and he finds that it is 80% of the bus fare for two persons. B finds that he has Rs. 3 with him and hands it over to A. In this context, which one of the following statements is correct ?

   (a) Now the money A has just enough to buy two tickers.

   (b) A still needs Rs. 2 for buying the tickets.

   (c) After buying the two tickets A will be left with 50 paise.

   (d) The money A now has is still not sufficient to buy two tickets.

   [CSAT 2014]

4. A gardener increased the area of his rectangular garden by increasing its length by 40% and decreasing its width by 20%. The area of the new garden

   (a) has increased by 20%.

   (b) has increased by 12%. **[CSAT 2014]**

   (c) has increased by 8%.

   (d) is exactly the same as the old area.

5. As per agreement with a bank, a businessman had to refund a loan in some equal installments without interest. After paying 18 installments he found that 60 percent of his loan was refunded. How many installments were there in the agreement ?

   (a) 22

   (b) 24

   (c) 30

   (d) 33 **[CSAT 2014]**

6. An automobile owner reduced his monthly petrol consumption when the prices went up. The price-consumption relationship is as follows:

   Price (in Rs. per litre) 40 50 60 75

   Monthly consumption (in litres) 60 48 40 32

   If the price goes up to Rs. 80 per litre, his expected consumption (in litres) will be

   (a) 30

   (b) 28

   (c) 26

   (d) 24 **[CSAT 2015]**

7. In a test, a candidate attempted only 8 questions and secured 50% marks in each of the questions. If the obtained a total of 40% in the test and all questions in the test carried equal marks, how many questions were there in the test ?

   (a) 8

   (b) 10

   (c) 15

   (d) 16 **[CSAT 2015]**

8. Candidates in a competitive examination consisted of 60% men and 40% women. 70% men and 75% women cleared the qualifying test and entered the final test where 80% men and 70% women were successful. Which of the following statements is correct ?

   (a) Success rate is higher for women.

   (b) Overall success rate in below 50%.

   (c) More men cleared the examination than women.

   (d) Both (a) and (b) above are correct.

   [CSAT 2015]

26. As a result of 25% hike in the price of rice per kg, a person is able to purchase 6 kg less rice for Rs. 1,200. What was the original price of rice per kg ?

    (a) Rs. 30   (b) Rs. 40

    (c) Rs. 50   (d) Rs. 60

    [CSAT 2020]

27. In a class, 60% of students are from India and 50% of the students are girls. If 30% of the Indian students are girls, then what percentage of foreign students are boys ?

    (a) 45%   (b) 40%

    (c) 30%   (d) 20%   **[CSAT 2021]**

28. A student appeared in 6 papers. The maximum marks are the same for each paper. His marks in these papers are in the proportion of 5 : 6 : 7 : 8 : 9 : 10. Overall he scored 60%. In how many number of papers did he score less than 60% of the maximum marks ?

    (a) 2   (b) 3

    (c) 4   (d) 5   **[CSAT 2021]**

29. P scored 40 marks more than Q in an examination. If Q scored 10% less marks than P, then how much did Q score ?

    (a) 360   (b) 380

    (c) 400   (d) 420   **[CSAT 2021]**

30. If the price of an article is decreased by 20% and then the new price is increased by 25%, then what is the net change in the price ?

    (a) 0%

    (b) 5% increase

    (c) 5% decrease

    (d) Cannot be determined due to insufficient data.

    [CSAT 2021]

31. The increase in the price of a certain item was 25%. Then the price was decreased by 20% and then again increased by 10%. What is the resultant increase in the price ?

    (a) 5%   (b) 10%

    (c) 12.5%   (d) 15%   **[CSAT 2022]**

32. When 70% of a number x is added to another number y, the sum becomes 165% of the value of y. When 60% of the number x is added to another number z, then the sum becomes 165% of the value of z. Which one of the following is correct ?

    (a) z < x < y   (b) x < y < z

    (c) y < x < z   (d) z < y < x

    [CSAT 2022]

33. Two candidates X and Y contested an election. 80% of voters cast their vote and there were no invalid votes. There was no NOTA (None of the above) option. X got 56% of the votes cast and won by 1440 votes. What is the total number of voters in the voters list ?

    (a) 15,000   (b) 12,000

    (c) 9,600   (d) 5,000 **[CSAT 2022]**

34. A principal P becomes Q in 1 year when compounded half-yearly with R% annual rate of interest. If the same principal P becomes Q in 1 year when compounded annually with S% annual rate of interest, then which one of the following is correct ?

    (a) $R = S$ (b) $R > S$

    (c) $R < S$ (d) $R \leq S$

    [CSAT 2023]

ANSWER KEY

1. (d)	6. (a)	11. (d)	16. (d)	21. (b)	26. (b)	31. (b)	33. (a)
2. (d)	7. (b)	12. (d)	17. (d)	22. (b)	27. (d)	32. (a)	34. (c)
3. (c)	8. (c)	13. (d)	18. (b)	23. (d)	28. (b)		
4. (b)	9. (c)	14. (b)	19. (a)	24. (c)	29. (a)		
5. (c)	10. (b)	15. (c)	20. (a)	25. (c)	30. (a)		

Profit & Loss

Terms related to Profit and Loss

Cost Price (CP) : The price at which an article is bought (purchased) is known as the *cost price*.

Selling Price (SP) : The price at which an article is sold is known as *selling price*.

Profit : If selling price is greater than cost price, then profit occurs.

Loss : If cost price is greater than selling price, then loss occurs.

Marked price / List price / MRP : The price which is printed on an article is known as Marked price / List price or MRP of the article.

Discount : Discount means the concession given to the customer on (MRP) marked price. (Discount is always given on marked price)

Note :

Profit or loss is always calculated with respect to cost price. (If we calculate profit or loss on selling price, it will be a mistake. However, sometimes questions are based on this concept also).

Mathematical Interpretation of Profit or Loss : If we sell an article at a profit of 10%, means we are selling the article at 110% of our cost price.

Similarly, if we are selling on article at 10% loss, means we are selling our article at 90% of cost price.

7.1 Based on CP and SP

1. If the cost price is Rs. 60 and profit percent is 20%. Find out the selling price.

Sol: Profit = 20% of Rs. 60 $= \dfrac{20 \times 60}{100}$ = Rs. 12

SP = CP + Profit = 60 + 12 = Rs. 72.

2. Find out the selling price, if cost price is Rs. 80 and loss percent is 15%.

Sol: Loss = 15% of Rs. 80 $= \dfrac{15 \times 80}{100}$ = Rs. 12

SP = CP – Loss = 80 – 12 = Rs. 68.

3. Find out the cost price, if selling price is Rs. 80 and profit percent is 25%.

Sol: Selling price = 125% of CP

According to the question,

125% of CP = Rs. 80

100% of CP $= \dfrac{80}{125} \times 100$ = Rs. 64.

Practice Set

Based on CP and SP

1. A man wanted to sell an article with 20% profit, but he actually sold at 20% loss for Rs. 480. At what price he wanted to sell it to earn the profit ?
   (a) Rs. 720
   (b) Rs. 840
   (c) Rs. 600
   (d) Rs. 750

2. By selling an article for Rs. 240, a man incurs a loss of 10%. At what price should he sell it, so that he makes a profit of 20% ?
   (a) Rs. 264
   (b) Rs. 288
   (c) Rs. 300
   (d) Rs. 320

3. By selling an article for Rs. 72, there is a loss of 10%. In order to gain 5%, its selling price should be
   (a) Rs. 87
   (b) Rs. 85
   (c) Rs. 80
   (d) Rs. 84

4. A man buys a cycle for Rs. 1400 and sells it at a loss of 15%. What is the selling price of the cycle ?
   (a) Rs. 1202
   (b) Rs. 1190
   (c) Rs. 1160
   (d) Rs. 1000

5. The ratio of cost price and selling price is 5 : 4, the loss percent is
   (a) 20%
   (b) 25%
   (c) 40%
   (d) 50%

6. If an article is sold for Rs. 178 at a loss of 11%, what should be its selling price in order to earn a profit of 11% ?
   (a) Rs. 222.50
   (b) Rs. 267
   (c) Rs. 435
   (d) Rs. 222

7. By selling a table for Rs. 350 instead of Rs. 400, loss percent increases by 5%. The cost price of table is
   (a) Rs. 1050
   (b) Rs. 417.50
   (c) Rs. 435
   (d) Rs. 1000

8. If selling price of an article is $\frac{8}{5}$ times its cost price, the profit percent on it is
   (a) 120%
   (b) 160%
   (c) 40%
   (d) 60%

9. On selling an article for Rs. 651, there is a loss of 7%. The cost price of that article is
   (a) Rs. 744
   (b) Rs. 751
   (c) Rs. 793
   (d) Rs. 700

10. The ratio of the CP and SP of an article is 20 : 21. What is the gain percent ?
    (a) 5%
    (b) 5.5%
    (c) 6%
    (d) 6.25%

11. In selling an article for Rs. 76, there is a profit of 52%. If it is sold for Rs. 75, the profit percent will be
    (a) 44
    (b) 46
    (c) 48
    (d) 50

12. The ratio of the cost price and selling price of an article is 5 : 6. What is the percentage of profit ?
    (a) 20%
    (b) 15%
    (c) 12.5%
    (d) 10%

13. Oranges are bought at 7 for Rs. 3. At what rate per hundred must they be sold to gain 33% ?
    (a) Rs. 56
    (b) Rs. 60
    (c) Rs. 58
    (d) Rs. 57

14. On selling an article for Rs. 105 a trader loses 9%. To gain 30% he should sell the article at
    (a) Rs. 126
    (b) Rs. 144
    (c) Rs. 150
    (d) Rs. 139

15. A merchant fixes the sale price of his goods at 15% above the cost price. He sells his goods at 12% less than the fixed price. His percentage of profit is

(a) $2\dfrac{1}{2}$ (b) $1\dfrac{1}{5}$

(c) $1\dfrac{1}{2}$ (d) 2

Defrauding

16. A dishonest grocer sells rice at a profit of 10% and also uses weights which are 20% less than the marked weight. The total gain earned by him will be

(a) 37.5%   (b) 40%

(c) 30.5%   (d) 35%

17. A dishonest dealer professes to sell his goods at the cost price but uses a false weight of 850 g instead of 1 kg. His gain percent is

(a) $17\dfrac{12}{17}\%$ (b) $17\dfrac{11}{17}\%$

(c) $71\dfrac{11}{17}\%$ (d) $11\dfrac{11}{17}\%$

18. A dishonest shopkeeper, using a faulty balance makes a profit of 5% while buying as well as while selling his goods. His actual gain percent in the whole process amounts to

(a) 11%

(b) 10%

(c) 10.25%

(d) 10.5%

19. A dishonest fruit vendor professes to sell his goods at a profit of 10% but he uses a weight of 16 gram for 20 gram. Find his gain percent

(a) 14%   (b) 24%

(c) 35%   (d) 37.5%

20. A cloth dealer professes to lose 20% on a certain garments, but he uses a metre having a length of 90 cm only and charges for the metre. Find his gain or loss percent

(a) $11\dfrac{1}{9}\%$ gain   (b) $11\dfrac{1}{9}\%$ loss

(c) 12.5% loss   (d) 12.5% gain

When the profit/loss is calculated on SP

21. 10% loss on selling price is what percent loss on the cost price ?

(a) $9\dfrac{1}{11}\%$ (b) $9\dfrac{2}{11}\%$

(c) 10%   (d) 11%

22. If loss is $\dfrac{1}{3}$ of SP, the loss percentage is

(a) $16\dfrac{2}{3}\%$

(b) 20%

(c) 25%

(d) 30%

$$\dfrac{x^2}{100}\%$$

23. A person sells two machines at Rs. 396 each. On one he gains 10% and on the other he loses 10%. His profit or loss in the whole transaction is

(a) No gain no loss   (b) 1% loss

(c) 1% profit   (d) 8% profit

24. A house and a shop were sold for Rs. 1 lakh each. In this transaction, the house sale resulted into 20% loss whereas the shop sale into 20% profit. The entire transaction resulted in

(a) no loss no gain

(b) gain of Rs. $\dfrac{1}{24}$ lakh

(c) loss of Rs. $\dfrac{1}{12}$ lakh

(d) loss of Rs. $\dfrac{1}{18}$ lakh

25. A shopkeeper sells two TV sets at the same price. There is a gain of 20% on one TV and loss of 20% on the other. State which of the following statement is correct ?

(a) The shopkeeper makes no net gain or profit

(b) The shopkeeper loses by 2%

(c) The shopkeeper gains by 4%

(d) The shopkeeper loses by 4%

38. A tradesman gives 4% discount on the marked price and gives 1 article free for buying every 15 articles and thus gains 35%. The marked price is above the cost price by

    (a) 20%   (b) 39%
    (c) 40%   (d) 50%

39. A trader marked his goods at 20% above the cost price. He sold half the stock at the marked price, one quarter at a discount of 20% on the marked price and the rest at a discount of 40% on the marked price. His total gain is

    (a) 2%   (b) 4.5%
    (c) 13.5%   (d) 15%

40. If books bought at prices ranging from Rs. 200 to Rs. 350 are sold at prices ranging from Rs. 300 to Rs. 425, what is the greatest possible profit that might be made in selling eight books ?

    (a) Rs. 400
    (b) Rs. 600
    (c) Cannot be determined
    (d) None of these

ANSWER KEY

1. (a)	6. (d)	11. (d)	16. (a)	21. (a)	26. (a)	31. (c)	36. (c)
2. (d)	7. (d)	12. (a)	17. (b)	22. (c)	27. (b)	32. (d)	37. (b)
3. (d)	8. (d)	13. (d)	18. (c)	23. (b)	28. (d)	33. (a)	38. (d)
4. (b)	9. (d)	14. (c)	19. (d)	24. (c)	29. (b)	34. (c)	39. (a)
5. (a)	10. (a)	15. (b)	20. (b)	25. (d)	30. (a)	35. (d)	40. (d)

Previous Year Questions

1. If Sohan, while selling two goats at the same price, makes a profit of 10% on one goat and suffers a loss of 10% on the other
   (a) he make no profit and no loss.
   (b) he makes a profit of 1%.
   (c) he suffers a loss of 1%.
   (d) he suffers a loss of 2%. **[CSAT 2014]**

2. A person allows a 10% discount for cash payment from the marked price of a toy and still he makes a 10% gain. What is the cost price of the toy which is marked Rs. 770 ?
   (a) Rs. 610   (b) Rs. 620
   (c) Rs. 630   (d) Rs. 640
   [CSAT 2016]

3. Gopal bought a cell phone and sold it to Ram at 10% profit. Then Ram wanted to sell it back to Gopal at 10% loss. What will be Gopal's position if he agreed ?
   (a) Neither loss nor gain
   (b) Loss 1%
   (c) Gain 1%
   (d) Gain 0.5% **[CSAT 2017]**

4. A shopkeeper sells an article at Rs. 40 and gets X% profit. However, when he sells it at Rs. 20, he faces same percentage of loss. What is the original cost of the article ?
   (a) Rs. 10   (b) Rs. 20
   (c) Rs. 30   (d) Rs. 40
   [CSAT 2018]

5. A person bought a car and sold it for Rs. 3,00,000. If he incurred a loss of 20%, then how much did he spend to buy the car ?
   (a) Rs. 3,60,000   (b) Rs. 3,65,000
   (c) Rs. 3,70,000   (d) Rs. 3,75,000
   [CSAT 2020]

ANSWER KEY

1. (c)   2. (c)   3. (c)   4. (c)   5. (d)

Chapter **8** Time, Speed and Distance

We all know,

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$x \text{ km/h} = \frac{5x}{18}\text{m/s} \text{ and } y \text{ m/s} = \frac{18y}{5}\text{km/h}$$

EXAMPLES

1. How many seconds does Aditya take to cover a distance of 400 m, if he runs at a speed of 20 km/h ?

Sol: Aditya's speed = 20 km/h $= \left(20 \times \frac{5}{18}\right)\text{m/s} = \frac{50}{9}\text{ m/s}$

Hence, time taken to cover 400 m $= \left(400 \times \frac{9}{50}\right)\text{s} = 72$ seconds.

2. A cyclist covers a distance of 750 m in 2 minutes 30 seconds. What is the speed in km/h of the cyclist ?

Sol: Time = 2 minutes 30 seconds = 150 seconds.

Speed $= \frac{750}{150}$ m/s $= 5$ m/s $= 5 \times \frac{18}{5}$ km/h $= 18$ km/h.

3. A man in a train notices that he can count 21 telephone posts in one minute. If they are known to be 50 metres apart, then at what speed is the train travelling ?

Sol: We know that, for n poles $(n - 1)$ intervals will be required.

So, total distance covered by the train in 1 minute = 20 × 50 = 1000 meter = 1 km

Train covers 1 km in 1 minute so in 60 minutes the train goes 60 km.

Hence, speed of the train = 60 km/h.

$$\textbf{Average speed} = \frac{\textbf{Total distance}}{\textbf{Total time}}$$

4. A man covered first 50 km at 25 km/h, next 30 km at 5 km/h and last 20 km at 10 km/h. Find out the average speed during the entire journey ?

Sol:

50 km	30 km	20 km
25 km/h	5 km/h	10 km/h

Time taken to cover 50 km $= \frac{50}{25} = 2$ hours

Time taken to cover 30 km $= \frac{30}{5} = 6$ hours

Practice Set

1. A thief is noticed by a policeman from a distance of 200 m. The thief starts running and the policeman chases him. The thief and the policeman run at the rate of 10 km and 11 km per hour respectively. What is the distance between them after 6 minutes ?
   - (a) 100 m
   - (b) 190 m
   - (c) 200 m
   - (d) 150 m

2. A and B start at the same time with speeds of 40 kmph and 50 kmph respectively. If in covering the journey A takes 15 minutes longer than B, the total distance of the journey is
   - (a) 46 km
   - (b) 48 km
   - (c) 50 km
   - (d) 52 km

3. A train running at $\frac{7}{11}$ of its own speed reached a place in 22 hours. How much time could be saved if the train would run at its own speed ?
   - (a) 14 hours
   - (b) 7 hours
   - (c) 8 hours
   - (d) 16 hours

4. A man can reach a certain place in 30 hours. If he reduces his speed by $\frac{1}{15}$ th, he goes 10 km less in that time. Find his speed per hour.
   - (a) 6 km/hr
   - (b) 5.5 km/hr
   - (c) 4 km/hr
   - (d) 5 km/hr

5. A 120 m long train takes 10 seconds to cross a man standing on a platform. What is the speed of the train ?
   - (a) 12 m/s
   - (b) 10 m/s
   - (c) 15 m/s
   - (d) 20 m/s

6. A man with $\frac{3}{5}$ of his usual speed reaches the destination 2.5 hours late. Find his usual time to teach the destination.
   - (a) 4 hours
   - (b) 3 hours
   - (c) $3\frac{3}{4}$ hours
   - (d) $4\frac{1}{2}$ hours

7. A student rides on bicycle at 8 km/h and reaches his school 2.5 minutes late. The next day he increases his speed to 10 km/h and reaches school 5 minutes early. How far is the school from his house ?
   - (a) $\frac{5}{8}$ km
   - (b) 8 km
   - (c) 5 km
   - (d) 10 km

8. A constable is 114 metres behind a thief. The constable runs 21 metres and the thief 15 metres in a minute. In what time will the constable catch the thief ?
   - (a) 19 minutes
   - (b) 18 minutes
   - (c) 17 minutes
   - (d) 16 minutes

9. Starting from his house one day, a student walks at a speed of 2.5 km/h and reaches his school 6 minutes late. Next day at the same time he increases his speed by 1 km/h and reaches the school 6 minutes early. How far is the school from his house ?
   - (a) 2 km
   - (b) $1\frac{1}{2}$ km
   - (c) 1 km
   - (d) $1\frac{3}{4}$ km

10. A car moves at the speed of 80 km/h. What is the speed of the car in metres per second ?
    - (a) 8 m/s
    - (b) $20\frac{1}{9}$ m/s
    - (c) $22\frac{2}{9}$ m/s
    - (d) None of these

11. Which of the following trains is the fastest ?
    - (a) 25 m/s
    - (b) 1500 m/min
    - (c) 90 km/h
    - (d) None of these

12. A person crosses a 600 m long street in 5 minutes. What is his speed in km per hour ?
    - (a) 3.6
    - (b) 7.2
    - (c) 8.4
    - (d) 10

ungist

1. (a)	5. (a)	9. (d)	13. (d)	17. (d)	19. (d)	21. (a)	23. (c)
2. (c)	6. (c)	10. (c)	14. (c)	18. (d)	20. (b)	22. (c)	24. (c)
3. (c)	7. (c)	11. (d)	15. (c)				
4. (d)	8. (a)	12. (b)	16. (d)				

Previous Year Questions

1. If a bus travels 160 km in 4 hours and a train travels 320 km in 5 hours at uniform speeds, then what is the ratio of the distances travelled by them in one hour ?
   (a) 8 : 5
   (b) 5 : 8
   (c) 4 : 5
   (d) 1 : 2   **[CSAT 2011]**

2. Mr. Kumar drives to work at an average speed of 48 km/h. The time taken to cover the first 60% of the distance is 10 minutes more than the time taken to cover the remaining distance. How far is his office ?
   (a) 30 km
   (b) 40 km
   (c) 45 km
   (d) 48 km   **[CSAT 2012]**

3. A thief running at 8 km/hr is chased by a policeman who speed is 10 km/hr. If the thief is 100 m ahead of the policeman, then the time required for the policemen to catch the thief will be
   (a) 2 minutes
   (b) 3 minutes
   (c) 4 minutes
   (d) 6 minutes   **[CSAT 2013]**

4. A person can walk a certain distance and drive back in six hours. He can also walk both ways in 10 hours. How much time will he take to drive both ways ?
   (a) Two hours
   (b) Two and a half hours
   (c) Five and a half hours
   (d) Four hours   **[CSAT 2013]**

5. A worker reaches his factory 3 minutes late if his speed from his house to the factory is 5 km/hr. If he walks at a speed of 6 km/hr, then he reaches the factory 7 minutes early the distance of the factory from his house is
   (a) 3 km
   (b) 4 km
   (c) 5 km
   (d) 6 km   **[CSAT 2014]**

6. Two cars start towards each other, from two places A and B which are at a distance of 160 km. They start at the same time 08 : 10 am. If the speeds of the cars are 50 km and 30 km/h respectively, they will meet each other at
   (a) 10 : 10 am
   (b) 10 : 30 am
   (c) 11 : 10 am
   (d) 11 : 20 am   **[CSAT 2014]**

7. In a 500 metres race, B starts 45 meters ahead of A, but A wins the race while B is still 35 metres behind. What is the ratio of the speed of A to B assuming that both start at the same time ?
   (a) 25 : 21
   (b) 25 : 20
   (c) 5 : 3
   (d) 5 : 7   **[CSAT 2015]**

8. Two cities A and B are 360 km apart. A car goes from A to B with a speed of 40 km/hr and returns to A with a speed of 60 km/hr. What is the average speed of the car ?
   (a) 45 km/hr
   (b) 48 km/hr
   (c) 50 km/hr
   (d) 55 km/hr
   [CSAT 2015]

9. A daily train is to be introduced between station A and station B starting from each end at 6 am and the journey is to be completed in 42 hours. What is the number of trains needed in order to maintain the shuttle service ?
   (a) 2
   (b) 3
   (c) 4
   (d) 7   **[CSAT 2016]**

10. A and B walk around a circular park. They start at 8 am from the same point in the opposite directions. A and B walk at a speed of 2 rounds per hour and 3 rounds per hour respectively. How many times shall they cross each other after 8:00 am and before 9:30 am ?
    (a) 7
    (b) 6
    (c) 5
    (d) 8   **[CSAT 2016]**

Chapter **9** **Trains**

9.1 Concept

- When a train passes a Person / Pole / Tree, train covers the distance which is equal to it's own length (since the length of the Person / Pole / Tree is negligible in comparison to the length of the train).

- When a train passes a Platform / Bridge / Tunnel, train covers a distance which is equal to it's own length + the length of the Platform.

EXAMPLES

1. A train 100 m long is running at the speed of 30 km/h. Find the time taken by it to pass a man standing near the railway line.

Sol: Speed of the train $= \left(30 \times \dfrac{5}{18} \right)$ m/s $= \left(\dfrac{25}{3} \right)$ m/s

Distance travelled in passing the standing man = 100 m

Hence, required time taken $= \dfrac{100}{\left(\dfrac{25}{3} \right)} = \left(100 \times \dfrac{3}{25} \right) = 12$ seconds.

2. A train is running at a speed of 132 km/h. If the length of the train is 110 metres, how long will it take to cross a railway bridge 165 metres long ?

Sol: Speed of train $= \left(132 \times \dfrac{5}{18} \right)$ m/s $= \left(\dfrac{110}{3} \right)$ m/s

Distance covered in passing the railway bridge = (110 + 165) m = 275 m

Hence, $\left(275 \times \dfrac{3}{110} \right) = \dfrac{15}{2} = 7\dfrac{1}{2}$ seconds.

3. A train 150 m long is running with a speed of 68 km/h. In what time will it pass a man who is running at 8 km/h in the same direction in which the train is going ?

Sol: Speed of the train relative to man = (68 – 8) km/h $= \left(60 \times \dfrac{5}{18} \right)$ m/s $= \left(\dfrac{50}{3} \right)$ m/s

Time taken by the train to cross the man

= Time taken by it to cover 150 m at $\left(\dfrac{50}{3} \right)$ m/s $= \left(150 \times \dfrac{3}{50} \right)$ seconds = 9 seconds.

Practice Set

1. A train passes a 50 metres long platform in 14 seconds and a man standing on the platform in 10 seconds. The speed of the train is
   (a) 24 km/h   (b) 36 km/h
   (c) 40 km/h   (d) 45 km/h

2. A train passes two bridges of lengths 800 m and 400 m in 100 seconds and 60 seconds respectively. The length of the train is
   (a) 80 m   (b) 90 m
   (c) 200 m   (d) 150 m

3. A 150 metre long train crosses a 500 metre long bridge in 30 seconds. What time will it take to cross a platform 370 metre long ?
   (a) 36 seconds   (b) 30 seconds
   (c) 24 seconds   (d) 18 seconds

4. A train moves past a telegraph post and a bridge 264 m long in 8 seconds and 20 seconds respectively. What is the speed of the train ?
   (a) 69.5 km/h
   (b) 70 km/h
   (c) 79 km/h
   (d) 79.2 km/h

5. A speed of 14 metres per second is the same as
   (a) 28 km/h
   (b) 46.6 km/h
   (c) 50.4 km/h
   (d) 70 km/h

6. A train 280 m long, running with a speed of 63 km/h will pass a tree in
   (a) 15 seconds
   (b) 16 seconds
   (c) 18 seconds
   (d) 20 seconds

7. A train 360 m long is running at a speed of 45 km/h. In what time will it pass a bridge 140 m long ?
   (a) 40 seconds   (b) 42 seconds
   (c) 45 seconds   (d) 48 seconds

8. A train running at the speed of 60 km/h crosses a pole in 9 seconds. What is the length of the train ?
   (a) 120 metres
   (b) 180 metres
   (c) Cannot be determined
   (d) None of these

9. A train covers a distance of 12 km in 10 minutes. If it takes 6 seconds to pass a telegraph post, then the length of the train is
   (a) 90 m   (b) 100 m
   (c) 120 m   (d) 140 m

10. The length of the bridge, which a train 130 metres long and travelling at 45 km/h can cross in 30 seconds, is
    (a) 200 m   (b) 225 m
    (c) 245 m   (d) 250 m

11. A goods train runs at the speed of 72 km/h and crosses a 250 m long platform in 26 seconds. What is the length of the goods train ?
    (a) 230 m
    (b) 240 m
    (c) 260 m
    (d) 270 m

12. A train of length 150 metres takes 40.5 seconds to cross a tunnel of length 300 metres. What is the speed of the train in km/hr ?
    (a) 13.33
    (b) 26.67
    (c) 40
    (d) 66.67

13. A train passes a station platform in 36 seconds and a man standing on the platform in 20 seconds. If the speed of the train is 54 km/hr, what is the length of the platform ?
    (a) 120 m   (b) 240 m
    (c) 300 m   (d) None of these

Previous Year Questions

1. A train travels at a certain average speed for a distance of 63 km and then travels a distance of 72 km at an average speed of 6 km/h more than its original speed. If it takes 3 hours to complete the total journey, what is the original speed of the train in km/h ?

    (a) 24   (b) 33

    (c) 42   (d) 66

    [CSAT 2013]

2. A freight train left Delhi for Mumbai at an average speed of 40 km/h. Two hours later, an express train left Delhi for Mumbai, following the freight train on a parallel track at an average speed of 60 km/h. How far from Delhi would the express train meet the freight train ?

    (a) 480 km   (b) 260 km

    (c) 240 km   (d) 120 km

    [CSAT 2017]

3. A train 200 metres long is moving at the rate of 40 km/h. In how many seconds will it cross a man standing near the railway line?

    (a) 12

    (b) 15

    (c) 16

    (d) 18

    [CSAT 2018]

ANSWER KEY

1. (c)   2. (c)   3. (d)

Chapter 10 — Boats & Streams

Upstream (U) : To row the boat against the flow.

Downstream (D) : To row the boat with the flow.

Let the speed of the boat in still water = 'B'.

Let the speed of the stream (current) = 'S'.

So, D = B + S   ...(1)

U = B – S   ...(2)

Solving equations (1) and (2), we get

$$B = \frac{D+U}{2} \ \text{ and } \ S = \frac{D-U}{2}$$

EXAMPLES

1. A man can row upstream at 7 km/h and downstream at 10 km/h. Find man's rate in still water and the rate of current.

Sol: Rate in still water $= \frac{1}{2}(10+7)$ km/h $= 8.5$ km/h

Rate of current $= \frac{1}{2}(10-7)$ km/h $= 1.5$ km/h

2. A man takes 3 hours 45 minutes to row a boat 15 km downstream of a river and 2 hours 30 minutes to cover a distance of 5 km upstream. Find the speed of the river current in km/h.

Sol: Rate downstream $= \left(\dfrac{15}{3\frac{3}{4}}\right)$ km/h $= \left(15 \times \dfrac{4}{15}\right)$ km/h $= 4$ km/h

Rate upstream $= \left(\dfrac{5}{2\frac{1}{2}}\right)$ km/h $= \left(5 \times \dfrac{2}{5}\right)$ km/h $= 2$ km/h

Speed of the river current $= \dfrac{4-2}{2}$ km/h $= 1$ km/h.

1. In one hour, a boat goes 11 km along the stream and 5 km against the stream. The speed of the boat in still water (in km/h) is
   (a) 3   (b) 5
   (c) 8   (d) 9

2. A man can row upstream at 8 km/h and downstream at 13 km/h. The speed of the stream is
   (a) 2.5 km/h   (b) 4.2 km/h
   (c) 5 km/h   (d) 10.5 km/h

3. A man rows downstream 32 km and 14 km upstream. If he takes 6 hours to cover each distance, then the velocity (in km/h) of the current is
   (a) $\dfrac{1}{2}$ (b) 1
   (c) $1\dfrac{1}{2}$ (d) 2

4. A boat running downstream covers a distance of 16 km in 2 hours while for covering the same distance upstream, it takes 4 hours. What is the speed of the boat in still water ?
   (a) 4 km/h   (b) 6 km/h
   (c) 8 km/h   (d) Data inadequate

5. A boatman goes 2 km against the current of the stream in 1 hour and goes 1 km along the current in 10 minutes. How long will it take to go 5 km in stationary water ?
   (a) 40 minutes   (b) 1 hour
   (c) 1 hr 15 min   (d) 1 hr 30 min

6. A man takes half time in rowing a certain distance downstream than upstream. What is the ratio of the speed in still water to the speed of current ?
   (a) 1 : 2   (b) 2 : 1
   (c) 1 : 3   (d) 3 : 1

7. A man takes twice as long to row a distance against the stream as to row the same distance in favour of the stream. The ratio of the speed of the boat (in still water) and the stream is
   (a) 2 : 1   (b) 3 : 1
   (c) 3 : 2   (d) 4 : 3

8. A boat running upstream takes 8 hours 48 minutes to cover a certain distance, while it takes 4 hours to cover the same distance running downstream. What is the ratio between the speed of the boat and speed of the water current respectively ?
   (a) 2 : 1
   (b) 3 : 2
   (c) 8 : 3
   (d) None of these

9. If a boat goes 7 km upstream in 42 minutes and the speed of the stream is 3 km/h, then the speed of the boat in still water is
   (a) 4.2 km/h   (b) 9 km/h
   (c) 13 km/h   (d) 21 km/h

10. A man's speed with the current is 15 km/h and the speed of the current is 2.5 km/h. The man's speed against the current is
    (a) 8.5 km/h   (b) 9 km/h
    (c) 10 km/h   (d) 12.5 km/h

11. If a man rows at the rate of 5 km/h in still water and his rate against the current is 3.5 km/h, then the man's rate along the current is
    (a) 4.25 km/h   (b) 6 km/h
    (c) 6.5 km/h   (d) 8.5 km/h

12. A boat can travel with a speed of 13 km/h in still water. If the speed of the stream is 4 km/h, find the time taken by the boat to go 68 km downstream
    (a) 2 hours   (b) 3 hours
    (c) 4 hours   (d) 5 hours

Chapter 11 Time and Work

This is one of the very much logical and practical topics of Quantitative aptitude. The approach that we are going to follow here, is LCM approach, and this LCM will be considered as the total work.

EXAMPLES

1. A does a work in 10 days and B does the same work in 15 days. In how many days they together will do the same work ?

Sol: A = 10 days — 3

B = 15 days — 2 $\Big\} 30$

Total work = LCM of (3, 2) = 30 units

A is doing $\dfrac{30}{10}$ = 3 units in 1 day

B is doing $\dfrac{30}{15}$ = 2 units in 1 day

Together they are doing = 3 + 2 = 5 units in 1 day.

Hence, time taken by them to complete the total work, working together $= \dfrac{30}{5} = 6$ days.

2. Worker A can complete a piece of work in 8 hours and B in 10 hours. In how many hours both of them working together will complete it ?

Sol: A = 8 hours — 5

B = 10 hours — 4 $\Big\} 40$

Total work = LCM of (8, 10) = 40 units

A is doing $\dfrac{40}{8}$ = 5 units in 1 hour

B is doing $\dfrac{40}{10}$ = 4 units in 1 hour

Together they are doing = 5 + 4 = 9 units in 1 hour

Hence, time taken by them to complete the total work, working together $= \dfrac{40}{9} = 4\dfrac{4}{9}$ hours.

46. A man, a woman and a boy can complete a job in 3, 4 and 12 days respectively. How many boys must assist 1 man and 1 woman to complete the job in $\frac{1}{4}$ th of a day ?

Sol: Man = 3 days $\searrow$ 4

Women = 4 days $\xrightarrow{3}$ 12

Boy = 12 days $\nearrow$ 1

If they complete the job in $\frac{1}{4}$ th of a day, they can complete 4 times of the work in 1 day.

So, 1M + 1W + nB = 48

4 + 3 + nB = 48

nB = 41

Putting B = 1, we get n = 41

So, 41 boys must assist 1 man and 1 woman to complete the job in $\frac{1}{4}$ th of a day.

47. 1 man or 3 woman or 4 boys can do a work in 38 days. Then in how many days will 1 man, 1 women and 1 boy do the work ?

Sol: 1M = 3W = 4B

1M = 4B and 1W $= \frac{4}{3}$ B

So, 1M + 1W + 1B = 4B + $\frac{4}{3}$ B + 1B = 5B + $\frac{4}{3}$ B = $\frac{19}{3}$ B

On using $M_1 D_1 = M_2 D_2$, we get

$38 \times 4 = \frac{19}{3} \times D_2$

Hence, 24 days.

Practice Set

Basic introduction

1. A and B can do a work in 12 days, B and C in 15 days and C and A in 20 days. If A, B and C work together, they will complete the work in

   (a) 5 days   (b) $7\frac{5}{6}$ days

   (c) 10 days   (d) $15\frac{2}{3}$ days

2. A and B can together finish a work in 30 days. They worked together for 20 days and then B left. After another 20 days, A finished the remaining work. In how many days A alone can finish the job ?

   (a) 50   (b) 60

   (c) 48   (d) 54

3. A and B can do a piece of work in 12 days, B and C in 8 days and C and A in 6 days. How long would B take to do the same work alone ?

   (a) 24 days   (b) 32 days

   (c) 40 days   (d) 48 days

4. A, B and C can complete a work separately in 24, 36 and 48 days respectively. They started together but C left after 4 days of start and A left 3 days before the completion of the work. In how many days will the work be completed ?

   (a) 15 days   (b) 22 days

   (c) 25 days   (d) 35 days

5. A man can do a job in 15 days. His father takes 20 days and his son finishes it in 25 days. How long will they take to complete the job if they all work together ?

   (a) Less than 6 days

   (b) Exactly 6 days

   (c) Approximately 6.4 days

   (d) More than 10 days

6. A and B can do a work in 8 days, B and C can do the same work in 12 days. A, B and C together can finish it in 6 days. A and C together will do it in

   (a) 4 days   (b) 6 days

   (c) 8 days   (d) 12 days

7. A and B can do a piece of work in 5 days; B and C can do it in 7 days; A and C can do it in 4 days. Who among these will take the least time if put to do it alone ?

   (a) A   (b) B

   (c) C   (d) Data inadequate

8. A can do a work in 15 days and B in 20 days. If they work on it together for 4 days, then the fraction of the work that is left is

   (a) $\frac{1}{4}$ (b) $\frac{1}{10}$

   (c) $\frac{7}{15}$ (d) $\frac{8}{15}$

9. A and B can complete a work in 15 days and 10 days respectively. They started doing the work together but after 2 days B had to leave and A alone completed the remaining work. The whole work was completed in

   (a) 8 days   (b) 10 days

   (c) 12 days   (d) 15 days

10. A machine P can print one lakh books in 8 hours, machine Q can print the same number of books in 10 hours while machine R can print them in 12 hours. All the machines are started at 9 am while machine P is closed at 11 am and the remaining two machines complete the work. Approximately at what time will the work be finished ?

    (a) 11:30 am   (b) 12 noon

    (c) 12:30 pm   (d) 1 pm

33. A, B and C can do a piece of work in 11 days, 20 days and 55 days respectively, working alone. How soon can the work be done if A is assisted by B and C on alternate days ?

    (a) 7 days   (b) 8 days

    (c) 9 days   (d) 10 days

34. 10 men can complete a piece of work in 15 days and 15 women can complete the same work in12 days. If all the 10 men and 15 women work together, in how many days will the work get completed ?

    (a) 6   (b) $6\dfrac{1}{3}$

    (c) $6\dfrac{2}{3}$ (d) $7\dfrac{2}{3}$

35. If 5 men or 8 women can do a piece of work in 12 days, how many days will be taken by 2 men and 4 women to do the same work ?

    (a) 15 days   (b) $13\dfrac{1}{2}$ days

    (c) $13\dfrac{1}{3}$ days   (d) 10 days

36. 6 men or 12 women can do a piece of work in 20 days. In how many days can 8 men and 16 women do twice as big as this work ?

    (a) 2 days   (b) 5 days

    (c) 15 days   (d) 10 days

37. 3 men or 5 women can do a work in 12 days. How long will 6 men and 5 women take to finish the work ?

    (a) 20 days   (b) 10 days

    (c) 4 days   (d) 15 days

ANSWER KEY

1. (c)	6. (c)	11. (b)	16. (c)	21. (c)	26. (b)	31. (d)	36. (c)
2. (b)	7. (a)	12. (c)	17. (c)	22. (b)	27. (b)	32. (b)	37. (c)
3. (d)	8. (d)	13. (a)	18. (b)	23. (b)	28. (a)	33. (b)	
4. (a)	9. (c)	14. (b)	19. (b)	24. (b)	29. (a)	34. (c)	
5. (c)	10. (d)	15. (b)	20. (a)	25. (b)	30. (a)	35. (c)	

Previous Year Questions

1. In a garrison, there was food for 1000 soldiers for one month. After 10 days, 1000 more soldiers joined the garrison. How long would the soldiers be able to carry on with the remaining food ?
   (a) 25 days
   (b) 20 days
   (c) 15 days
   (d) 10 days

   [CSAT 2013]

2. Ram and Shyam work on a job together for four days and complete 60% of it. Ram takes leave then and Shyam works for eight more days to complete the job. How long would Ram take to complete the entire job alone ?
   (a) 6 days
   (b) 8 days
   (c) 10 days
   (d) 11 days

   [CSAT 2016]

3. W can do 25% of a work in 30 days, X can do $\frac{1}{4}$ of the work in 10 days, Y can do 40% of the work in 40 days and Z can do $\frac{1}{3}$ of the work in 13 days. Who will complete the work first ?
   (a) W
   (b) X
   (c) Y
   (d) Z   **[CSAT 2016]**

4. P works thrice as fast as Q, whereas P and Q together can work four times as fast as R. If P, Q and R together work on a job, in what ratio should they share the earnings ?
   (a) 3 : 1 : 1
   (b) 3 : 2 : 4
   (c) 4 : 3 : 4
   (d) 3 : 1 : 4

   [CSAT 2017]

5. A man completes $\frac{7}{8}$ of a job in 21 days. How many more days will it take him to finish the job if quantum of work is further increased by 50% ?
   (a) 24
   (b) 21
   (c) 18
   (d) 15   **[CSAT 2021]**

6. 24 men and 12 women can do a piece of work in 30 days. In how many days can 12 men and 24 women do the same piece of work ?
   (a) 30 days
   (b) More than 30 days
   (c) Less than 30 days or more than 30 days
   (d) Data is inadequate to draw any conclusion

   [CSAT 2022]

7. *A*, *B*, *C* working independently can do a piece of work in 8, 16 and 12 days respectively. *A* alone works on Monday, *B* alone works on Tuesday, *C* alone works on Wednesday; *A* alone, again works on Thursday and so on. Consider the following statements :
   1. The work will be finished on Thursday.
   2. The work will be finished in 10 days.
   Which of the above statements is/are correct ?
   (a) 1 only
   (b) 2 only
   (c) Both 1 and 2
   (d) Neither 1 nor 2

   [CSAT 2023]

ANSWER KEY

1. (d)	2. (c)	3. (d)	4. (a)	5. (d)	6. (d)	7. (a)

Chapter 12 — **Pipes** and **Cisterns**

This topic is the continuation of Time and work only, the approach of solving questions is similar to that of the last topic. The only difference here is that the HCF would be considered as the capacity of the tank.

EXAMPLES

1. Two pipes A and B can fill a tank in 36 hours and 45 hours respectively. If both the pipes are opened simultaneously, how much time will be taken to fill the tank ?

Sol: $A = 36$ hours $\searrow 5$

$\qquad\qquad\qquad\qquad > 180$

$\quad B = 45$ hours $\nearrow 4$

Total work = LCM of (36, 45) = 180 units

A is doing $\dfrac{180}{36} = 5$ units in 1 hour and B is doing $\dfrac{180}{45} = 4$ units in 1 hour

Together they are doing = 5 + 4 = 9 units in 1 hour.

Hence, time taken by them to complete the total work, working together $= \dfrac{180}{9} = 20$ hours.

2. Two pipes can fill a tank in 10 hours and 12 hours respectively while a third pipe empties the full tank in 20 hours. If all the three pipes operate simultaneously, in how much time will the tank be filled ?

Sol: $A = 10$ hours $\searrow 6$

$\quad B = 12$ hours $\xrightarrow{5} 60$

$\quad C = 20$ hours $\nearrow -3$
$\qquad$ (Leak)

Total work = LCM of (10, 12, 20) = 60 units

A is doing $\dfrac{60}{10} = 6$ units in 1 hour and B is doing $\dfrac{60}{12} = 5$ units in 1 hour

Here, C is a leak pipe. So, it's work will be treated as negative work.

C is doing $\dfrac{60}{20} = -3$ units in 1 hour

Together they are doing = 6 + 5 − 3 = 8 units in 1 hour.

Hence, time taken by them to complete the total work, working together $= \dfrac{60}{8} = 7.5$ hours.

Practice Set

1. Two pipes A and B can fill a tank in 20 minutes and 30 minutes respectively. If both pipes are opened together, the time taken to fill the tank is
   (a) 50 minutes
   (b) 12 minutes
   (c) 25 minutes
   (d) 15 minutes

2. If $\frac{1}{3}$ of a tank holds 80 litres of water, then the quantity of water that $\frac{1}{2}$ tank holds is
   (a) 240 litres
   (b) 120 litres
   (c) $\frac{80}{3}$ litres
   (d) 100 litres

3. A cistern can be filled with water by a pipe in 5 hours and it can be emptied by a second pipe in 4 hours. If both the pipes are opened when the cistern is full the time in which it will be emptied is.
   (a) 9 hours
   (b) 18 hours
   (c) 20 hours
   (d) $20\frac{1}{2}$ hours

4. Two pipes A and B can separately fill a cistern in 60 minutes and 75 minutes respectively. There is a third pipe in the bottom of the cistern to empty it. If all the three pipes are simultaneously opened, then the cistern is full in 50 minutes. In how much time the third pipe alone can empty the cistern ?
   (a) 110 minutes
   (b) 100 minutes
   (c) 120 minutes
   (d) 90 minutes

5. Two pipes A and B can fill a cistern in $37\frac{1}{2}$ minutes and 45 minutes respectively. Both pipes are opened. The cistern will be filled just in half an hour, if the pipe B is turned off after
   (a) 15 minutes
   (b) 10 minutes
   (c) 5 minutes
   (d) 9 minutes

6. Two taps A and B can fill a tank in 48 minutes and 36 minutes, if both taps are opened together after how much time tap A is closed so that the whole tank fill in 25 minutes 30 seconds
   (a) 14 minutes
   (b) 21 minutes
   (c) 28 minutes
   (d) 35 minutes

7. Two pipes can fill a tank with water in 15 and 12 hours respectively, and a third pipe can empty it in 4 hours. If the pipes be opened in order, at 8, 9 and 11 am respectively, the tank will be emptied at
   (a) 11:40 am
   (b) 12:40 pm
   (c) 1:40 pm
   (d) 2:40 pm

8. Three pipes P, Q and R can separately fill a cistern in 4, 8 and 12 hours respectively. Another pipe S can empty the completely filled cistern in 10 hours. Which of the following arrangements will fill the empty cistern in less time than others ?
   (a) Q alone is open
   (b) P and S are open
   (c) P and Q are open
   (d) P, Q and S are open

9. One pipe fills a water tank three times faster than another pipe. If the two pipes together can fill the empty tank in 36 minutes, then how much time will the slower pipe alone take to fill the tank ?
   (a) 1 hour 21 minutes
   (b) 1 hour 48 minutes
   (c) 2 hours
   (d) 2 hour 24 minutes

10. One tap can fill a water tank in 40 minutes and another tap can make the filled tank empty in 60 minutes. If both the taps are open, in how many hours will the empty tank be filled ?
    (a) 2
    (b) 2.5
    (c) 3
    (d) 3.5

Previous Year Questions

1. Two pipes A and B can independently fill a tank completely in 20 and 30 minutes respectively. If both the pipes are opened simultaneously, how much time will they take to fill the tank completely ?

   (a) 10 minutes   (b) 12 minutes

   (c) 15 minutes   (d) 25 minutes

   [CSAT 2015]

2. There are three pillars X, Y and Z of different heights. Three spiders A, B and C start to climb on these pillars simultaneously. In one chance, A climbs on X by 6 cm but slips down 1 cm. B climbs on Y by 7 cm but slips down 3 cm. C climbs on Z by 6.5 cm but slips down 2 cm. If each of them requires 40 chances to reach the top of the pillars, what is the height of the shortest pillar ?

   (a) 161 cm   (b) 163 cm

   (c) 182 cm   (d) 210 cm

   [CSAT 2017]

3. A frog tries to come out of a dried well 4.5 m deep with slippery walls. Every time the frog jumps 30 cm, slides down 15 cm. What is the number of jumps required for the frog to come out of the well ?

   (a) 28   (b) 29

   (c) 30   (d) 31   **[CSAT 2020]**

ANSWER KEY

1. (b)   2. (b)   3. (b)

Chapter 13 — Permutations and Combinations

13.1 Basic Counting Principle

And $\rightarrow$ '$\times$'

Or $\rightarrow$ '$+$'

EXAMPLES

1. If a coin is tossed and a dice is thrown. Find out total number of all the possible outcomes ?

Sol: Total possible outcomes are given as following :

$$\left.\begin{array}{lcccccc}\text{Dice} & 1 & 2 & 3 & 4 & 5 & 6 \\ \text{Coin} & H & H & H & H & H & H\end{array}\right\} 6 \quad \text{and} \quad \left.\begin{array}{lcccccc}\text{Dice} & 1 & 2 & 3 & 4 & 5 & 6 \\ \text{Coin} & T & T & T & T & T & T\end{array}\right\} 6$$

Total $6 + 6 = 12$

Coin 'and' Dice

$2 \times 6 = 12$

Here 'and' means product.

2. A Mock Test contains 2 sections with 3 and 4 questions respectively. In how many ways can a student select one question from each sections ?

Sol:
$$\begin{array}{l|cccc}\text{Section A} & a_1 & a_2 & a_3 & \\ \hline \text{Section B} & b_1 & b_2 & b_3 & b_4\end{array}$$

Possible selection of questions

$(a_1 b_1), (a_1 b_2), (a_1 b_3), (a_1 b_4) = 4$

$(a_2 b_1), (a_2 b_2), (a_2 b_3), (a_2 b_4) = 4$

$(a_3 b_1), (a_3 b_2), (a_3 b_3), (a_3 b_4) = 4$

Total ways to select questions $= 4 + 4 + 4 = 12 = (3 \times 4)$

$\qquad\qquad\qquad$ = Questions in Section A $\times$ Questions in Section B.

3. There are 6 trains running from New Delhi to Bhopal. A man goes from New Delhi to Bhopal and comes back with a different train. In how many ways this can be done ?

Sol:

New Delhi	T_1	T_2	T_3	T_4	T_5	T_6
Bhopal	T_1	T_2	T_3	T_4	T_5	T_6

For onward journey man has 6 choices and for downward journey he has 5 choices.

So, total number of ways $= 6 \times 5 = 30$.

EXAMPLE

1. Find the unit digit of the given expression 1! + 2! + 3! + …… + 100! ?

Sol: We know that,

1! = 1, 2! = 2, 3! = 6, 4! = 24, 5! = 120, 6! = 720, 7! = 5040 and so on.

When they will be added up to 100!, unit digit will be 3.

13.3 $^{n}C_{r}$: Selection of r articles out of n articles.

Number of ways of selection of 'r' articles out of 'n' articles, where order does not matter.

$$^{n}C_{r} = \frac{n!}{r! \cdot (n-r)!}$$

$$^{n}C_{n-r} = \frac{n!}{(n-r)! \cdot r!}$$

So, we can say $^{n}C_{r} = \,^{n}C_{n-r}$

Note: $^{n}C_{0} = 1$, $^{n}C_{n} = 1$, $^{n}C_{1} = n$

EXAMPLES

$$^{5}C_{2} = \frac{5!}{2! \times 3!} = \frac{5 \times 4 \times 3!}{1 \times 2 \times 3!} = \frac{5 \times 4}{1 \times 2} = \frac{20}{2} = 10.$$

$$^{7}C_{2} = \frac{7!}{2! \times 5!} = \frac{7 \times 6 \times 5!}{1 \times 2 \times 5!} = \frac{7 \times 6}{1 \times 2} = \frac{42}{2} = 21.$$

$$^{7}C_{3} = \frac{7!}{3! \times 4!} = \frac{7 \times 6 \times 5 \times 4!}{1 \times 2 \times 3 \times 4!} = \frac{7 \times 6 \times 5}{1 \times 2 \times 3} = \frac{210}{6} = 35.$$

$$^{10}C_{3} = \frac{10!}{3! \times 7!} = \frac{10 \times 9 \times 8 \times 7!}{1 \times 2 \times 3 \times 7!} = \frac{10 \times 9 \times 8}{1 \times 2 \times 3} = \frac{720}{6} = 120.$$
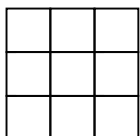
$$^{10}C_{4} = \frac{10!}{4! \times 6!} = \frac{10 \times 9 \times 8 \times 7 \times 6!}{1 \times 2 \times 3 \times 4 \times 6!} = \frac{10 \times 9 \times 8 \times 7}{1 \times 2 \times 3 \times 4} = \frac{5040}{24} = 210.$$

We can clearly observe, $^{n}C_{2} = \frac{n(n-1)}{2}$.

13.4 Total number of squares

☐ 1 square

 $\Rightarrow \begin{array}{l} 1 \times 1 = 4 \\ 2 \times 2 = 1 \end{array}$ $\Rightarrow 1 + 4 = 1^{2} + 2^{2} = 5$ squares.

 $\Rightarrow \begin{array}{l} 1 \times 1 = 9 \\ 2 \times 2 = 4 \\ 3 \times 3 = 1 \end{array}$ $\Rightarrow 1 + 4 + 9 = 1^{2} + 2^{2} + 3^{2} = 14$ squares.

EXAMPLES

1. Find out the total number rectangles in the given figure ?
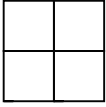
Sol: Here, 3 horizontal parallel lines are intersecting another set of 3 vertical parallel lines, so total number of rectangle will be

$$\Rightarrow {}^3C_2 \times {}^3C_2 = 3 \times 3 = 9$$
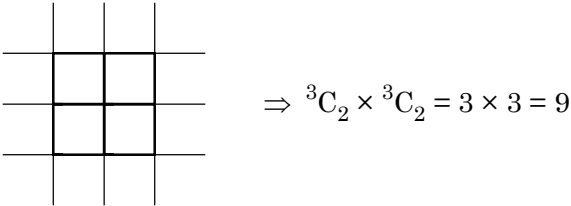
In the above figure if we calculate squares, there will be 5 squares, and if we calculate those rectangles which are not square, they will be 4. Hence, total number of rectangles will be 5 + 4 = 9.

Number of rectangles which are not square in the given figure = total rectangles – squares

$$= {}^3C_2 \times {}^3C_2 - \Sigma 2^2 = 9 - 5 = 4.$$

2. Find out the number of rectangle in the following figure which are not square.
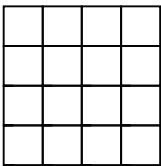
Sol: Number of rectangles which are not square in the given figure = total rectangles – squares

$$= {}^5C_2 \times {}^5C_2 - \Sigma 4^2 = 100 - 30 = 70.$$

Hence, there are 70 rectangles in the given figure which are not square.

13.6 Number of Integral Solutions

$$x_1 + x_2 + x_3 + \ldots\ldots + x_r = n$$

Where $x_1, x_2, x_3, x_4, \ldots\ldots, x_r$ and n are non-negative (≥ 0) integers

Total number of integral solutions = ${}^{n + r - 1}C_{r - 1}$.

EXAMPLES

1. Find out the integral number of solutions of the equation, $x + y = 6$, where $x, y \geq 0$.

Sol: Here, $r = 2$ and $n = 6$

Total number of integral solutions of the given equation = ${}^{6 + 2 - 1}C_{2 - 1} = {}^7C_1 = 7$.

2. 25 identical marbles are to be divided among four boys in such a way that none of them receive less than 3. In how many ways this can be done ?

Sol: First we will give 3 marbles to each boy and then we will randomly distribute the marbles among them. When we give 3 marbles to each boy means we are left with $25 - 12 = 13$ marbles now.

So, we will get the following equation.

$x + y + z + w = 13$, here, $x, y, z, w \geq 0$

Total number of integral solutions of the above equation = ${}^{13 + 4 - 1}C_{4 - 1} = {}^{16}C_3 = 560$.

235

Practice Set

1. How many words can be formed from the letters of the word SIGNATURE so that the vowels always come together ?
   (a) 720   (b) 1440
   (c) 2880   (d) None of these

2. In how many ways a committee consisting of 5 men and 6 women can be formed from 8 men and 10 women ?
   (a) 266   (b) 86400
   (c) 11760   (d) 5040

3. In how many different ways can the letters of the word DESIRE be rearranged in such a way that the vowels always come together ?
   (a) 24   (b) 72
   (c) 144   (d) 360

4. A team of 5 children is to be selected out of 4 girls and 5 boys such that it contains at least 2 girls. In how many different ways can be selection be made ?
   (a) 105   (b) 60
   (c) 100   (d) 120

5. In how many different ways can the letters of the word DIGEST be arranged so that the vowels do not come together ?
   (a) 720   (b) 240
   (c) 480   (d) 360

6. Letter of the word DIRECTOR are arranged in such a way that all the vowels come together. Find out the total number of ways for making such arrangement.
   (a) 4320   (b) 2720
   (c) 2160   (d) 1120

7. How many different letter arrangements can be made from the letters of the word RECOVER ?
   (a) 1210
   (b) 5040
   (c) 1260
   (d) 1200

8. 4 boys and 2 girls are to be seated in a row in such a way that the two girls are always together. In how many different ways can they be seated ?
   (a) 120   (b) 720
   (c) 148   (d) 240

9. On the occasion of a certain meeting each member gave shakehand to the remaining members. If the total shakehands were 28, how many members were present for the meeting ?
   (a) 14   (b) 7
   (c) 9   (d) 8

10. 8 men entered a lounge simultaneously. If each person shook hands with the other then find the total number of handshakes.
    (a) 16   (b) 36
    (c) 56   (d) 28

11. In how many different ways can the letter of the word JUDGE be arranged so that the vowels always come together ?
    (a) 48   (b) 24
    (c) 120   (d) 60

12. From a group of 6 men and 4 women a committee of 4 persons is to be formed. In how many different ways can it be done so that the committee has at least one woman ?
    (a) 210   (b) 225
    (c) 195   (d) 185

13. From a group of 6 men and 4 women a committee of 4 persons is to be formed. In how many different ways can it be done so that the committee has at least 2 men?
    (a) 210   (b) 225
    (c) 195   (d) 185

14. How many numbers between 1000 and 10000 contain the digits 1, 3, 5 and 7 ?
    (a) 16   (b) 24
    (c) 8   (d) 32

1. (d)	9. (d)	17. (d)	25. (b)	33. (d)	41. (b)	49. (b)	57. (a)
2. (c)	10. (d)	18. (d)	26. (b)	34. (b)	42. (c)	50. (c)	58. (b)
3. (b)	11. (a)	19. (d)	27. (b)	35. (b)	43. (b)	51. (a)	59. (b)
4. (a)	12. (c)	20. (d)	28. (c)	36. (b)	44. (d)	52. (b)	
5. (c)	13. (d)	21. (b)	29. (c)	37. (c)	45. (a)	53. (d)	
6. (c)	14. (b)	22. (a)	30. (b)	38. (c)	46. (d)	54. (d)	
7. (c)	15. (c)	23. (a)	31. (c)	39. (a)	47. (a)	55. (c)	
8. (d)	16. (c)	24. (a)	32. (b)	40. (c)	48. (c)	56. (b)	

Explanation of Selected Questions

26.
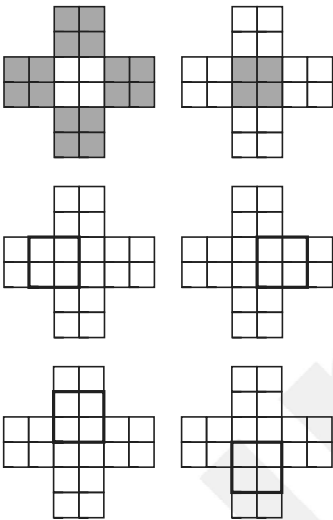
Total number of squares = $4 \times 5 + 5 + 1 + 1 + 1$ + 1 = 20 + 5 + 4 = 29.

27. The letters can be arranged as, VVC, VCV, CVV, CVC

(2 consonants are not together so CCC, CCV and VCC are not considered).

Ways of arranging VVC = $5 \times 5 \times 21 = 525$

Ways of arranging VCV = $5 \times 21 \times 5 = 525$

Ways of arranging CVV = $21 \times 5 \times 5 = 525$

Ways of arranging CVC = $21 \times 5 \times 21 = 2205$

So, total words

= 525 + 525 + 525 + 2205 = 3780.

28. There are 5 ingredients.

So, different salads can be made = using at least one ingredient at a time = $2^5 - 1 = 31$.

(Here, 2^5 is considered because every ingredient has 2 choices *i.e.* it may be used or it may not be)

29.
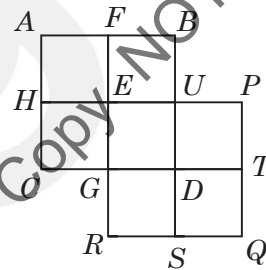
In figure *ABCD*, we have 9 quadrilaterals and in figure *PQRE*, we again have 9 quadrilaterals, but here *GDUE* is common in both.

So, number of rectangles = 9 + 9 − 1 = 17.

We have 2 more quadrilaterals, one is *RSBF* and other is *PTCH*.

So, total quadrilaterals = 17 + 2 = 19.

30. He can solve 4 questions from section 1 and 2 question from section 2, or 3 and 3 or 2 and 4 respectively.

A	B
a_1	b_1
a_2	b_2
a_3	b_3
a_4	b_4
a_5	b_5

A	B
2	4
3	3
4	2

Previous Year's Questions

1. With reference to the figure given below, the number of different routes from S to T without retracing from U and/or V, is
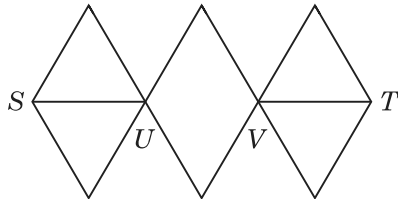
   (a) 3   (b) 6
   (c) 9   (d) 18   **[CSAT 2014]**

2. Twelve people form a club. By picking lots, one of them will host a dinner for all once in a month. The number of dinners a particular member has to host in one year is
   (a) One
   (b) Zero
   (c) Three
   (d) Cannot be predicated   **[CSAT 2015]**

3. A selection is to be made for one post of Principle and two posts of Vice-Principal. Amongst the six candidates called for the interview, only two are eligible for the post of Principle while they all are eligible for the post of Vice-Principal. The number of possible combinations of selectees is
   (a) 4
   (b) 12
   (c) 18
   (d) None of the above   **[CSAT 2015]**

4. There are 5 tasks and 5 persons. Task-1 cannot be assigned to either person-1 or person-2. Task-2 must be assigned to either person-3 or person-4. Every person is to be assigned one task. In how many ways can be assignment be done ?
   (a) 6
   (b) 12
   (c) 24
   (d) 144   **[CSAT 2015]**

5. In a question paper there are five questions to be attempted and answer to each question has two choices – True (T) or False (F). It is given that no two candidates have given the answer to the five questions in an identical sequence. For this to happen the maximum number of candidates is
   (a) 10
   (b) 18
   (c) 26
   (d) 32   **[CSAT 2016]**

6. 4-digit numbers are to be formed using the digits 1, 2, 3 and 4; and none of these four digits are repeated in any manner. Further,
   1. 2 and 3 are not to immediately follow each other.
   2. 1 is not to be immediately followed by 3.
   3. 4 is not to appear at the last place.
   4. 1 is not to appear at the first place.
   How many different numbers can be formed ?
   (a) 6
   (b) 8
   (c) 9
   (d) None of the above   **[CSAT 2016]**

7. There are 4 horizontal and 4 vertical lines, parallel and equidistant to one another on a board. What is the maximum number of rectangles and squares that can be formed ?
   (a) 16   (b) 24
   (c) 36   (d) 42   **[CSAT 2017]**

8. A bag contains 20 balls. 8 balls are green, 7 are white and 5 are red. What is the minimum number of balls that must be picked up from the bag blindfolded (without replacing any of it) to be *assured* of picking at least one ball of each colour ?
   (a) 17   (b) 16
   (c) 13   (d) 11   **[CSAT 2017]**

31. What is the number of selections of 10 consecutive things out of 12 things in a circle taken in the clockwise direction ?

(a) 3   (b) 11

(c) 12   (d) 66   **[CSAT 2023]**

32. In an examination, the maximum marks for each of the four papers namely P, Q, R and S are 100. Marks scored by the students are in integers. A student can score 99% in n different ways. What is the value of n ?

(a) 16   (b) 17

(c) 23   (d) 35   **[CSAT 2023]**

33. A flag has to be designed with 4 horizontal stripes using some or all of the colours red, green and yellow. What is the number of different ways in which this can be done so that no two adjacent stripes have the same colour ?

(a) 12   (b) 18

(c) 24   (d) 36   **[CSAT 2023]**

34. There are five persons P, Q, R, S and T each one of whom has to be assigned one task. Neither P nor Q can be assigned Task-1. Task-2 must be assigned to either R or S. In how many ways can the assignment be done ?

(a) 6   (b) 12

(c) 18   (d) 24   **[CSAT 2023]**

35. What is the sum of all 4-digit numbers less than 2000 formed by the digits 1, 2, 3 and 4, where none of the digits is repeated ?

(a) 7998

(b) 8028

(c) 8878

(d) 9238   **[CSAT 2023]**

ANSWER KEY

1. (d)	6. (a)	11. (a)	16. (b)	21. (c)	26. (b)	31. (c)	34. (d)
2. (d)	7. (c)	12. (d)	17. (b)	22. (c)	27. (b)	32. (d)	35. (a)
3. (d)	8. (b)	13. (d)	18. (c)	23. (d)	28. (c)	33. (c)	
4. (c)	9. (c)	14. (d)	19. (b)	24. (a)	29. (c)		
5. (d)	10. (c)	15. (b)	20. (c)	25. (d)	30. (c)		

Additional Questions from Other Competitive Exams

1. What is the number of distinct arrangements of the letters of the word UGCCSIR so that U and I cannot come together ?
   (a) 2520   (b) 720
   (c) 1520   (d) 1800

2. Suppose there are socks of N different colors in a box. If you take out one sock at a time, what is the maximum number of socks that you have to take out before a matching pair is found ? (assume that N is an even number)
   (a) N (b) $N+1$
   (c) $N-1$ (d) $\dfrac{N}{2}$

3. An ant goes from A to C in the figure crawling only on the lines and taking the least length of path. The number of ways in which it can do so is
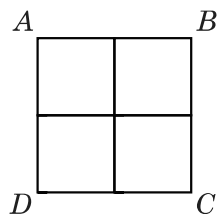
   (a) 2   (b) 4
   (c) 5   (d) 6

4. During a summer vacation, of 20 friends from a hostel, each wrote a letter to each of all others. The total number of letters written was
   (a) 20   (b) 400
   (c) 200   (d) 380

5. 366 players participate in a knock-out tournament. In each round all competing players pair together and play a match, the winner of each match moving to the next round. If at the end of a round there is an odd number of winners, the unpaired one moves to the next round without playing a match. What is the total number of matches played ?
   (a) 366   (b) 282
   (c) 365   (d) 418

6. A '4 × 4' magic square is given below :

1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16

   How many 2 × 2 squares are there in it whose elements add up to 34 ?
   (a) 6   (b) 9
   (c) 4   (d) 5

7. Suppose in a box there are 20 red, 30 black, 40 blue and 50 white balls. What is the minimum number of balls to be drawn, without replacement, so that you are certain about getting 4 red, 5 black, 6 blue and 7 white balls ?
   (a) 140   (b) 97
   (c) 104   (d) 124

8. From a group of 40 players, a cricket team of 11 players is chosen. Then, one of the eleven is chosen as the captain of the team. The total number of ways this can be done is [$\begin{pmatrix} m \\ n \end{pmatrix}$ below means the number of ways n objects can be chosen from m objects]
   (a) $^{40}C_{11}$ (b) $11 \times {}^{40}C_{11}$
   (c) $29 \times {}^{40}C_{11}$ (d) $^{39}C_{10}$

9. In how many distinguishable ways can the letters of the word CHANCE be arranged ?
   (a) 120   (b) 720
   (c) 360   (d) 240

10. In a group of 11 persons, each shakes hand with every other once and only once. What is the total number of such handshakes ?
    (a) 110
    (b) 121
    (c) 55
    (d) 66

Probability

Introduction

Probability is the chance of occurrence of an event, mathematically it can be represented as percentage or fraction. Maximum chance and minimum chance of an event can be 100% and 0 respectively.

Mathematical formula of probability is given by, $p(\text{E}) = \dfrac{n(\text{E})}{n(\text{S})}$

Here, $n(\text{S})$ is sample space and $n(\text{E})$ is the required outcome.

EXAMPLES

1. If a fair coin is tossed. Find the probability of getting a head ?

Sol: Required probability $= \dfrac{1}{2}$.

2. What will be the sample space in case of (i) 2 coins, (ii) 3 coins, (iii) 2 dice ?

Sol: Total outcomes of n coins $= 2^n$ and total outcomes of n dice $= 6^n$.

(i) Sample space in case of 2 coins $= 2^2 = 4$ i.e., (H, H), (H, T), (T, H) and (T, T)

(ii) Sample space in case of 3 coins $= 2^3 = 8$ i.e.,

$1 \to$ H H H
$2 \to$ H H T
$3 \to$ H T H
$4 \to$ H T T
$5 \to$ T H H
$6 \to$ T H T
$7 \to$ T T H
$8 \to$ T T T

(iii) Sample space in case of 2 dice $= 6^2 = 36$.

3. If 2 coins are tossed, find the probability of getting (i) at least 1 head, (ii) at most 2 tails, (iii) exactly 1 head.

Sol: Sample space in case of 2 coins $= 2^2 = 4$ i.e., (H, H), (H, T), (T, H) and (T, T)

(i) Probability of getting at least 1 head $= \dfrac{3}{4}$

(ii) Probability of getting at most 2 tails $= \dfrac{3}{4}$

(iii) Probability of getting exactly 1 head $= \dfrac{2}{4} = \dfrac{1}{2}$.

Practice Set

1. Ravi, Sunil and Maninder review a cricket book. Odds in favour of the book is 5 : 2, 4 : 3 and 3 : 4 respectively. Find the probability that majority are in favour of the book ?

   (a) $\dfrac{149}{343}$ (b) $\dfrac{60}{343}$

   (c) $\dfrac{209}{343}$ (d) $\dfrac{211}{343}$

2. The odds against an event is 7 : 3. Find the probability of the occurrence of the event ?

   (a) $\dfrac{7}{10}$ (b) $\dfrac{3}{10}$

   (c) $\dfrac{4}{10}$ (d) $\dfrac{7}{3}$

3. In a simultaneous throw of three dice find the probability of getting a total 5 ?

   (a) $\dfrac{1}{36}$ (b) $\dfrac{4}{36}$

   (c) $\dfrac{5}{36}$ (d) $\dfrac{6}{36}$

4. A coin is successively tossed three times. Find the probability of getting exactly one tail.

   (a) $\dfrac{2}{8}$ (b) $\dfrac{3}{8}$

   (c) $\dfrac{5}{8}$ (d) $\dfrac{6}{8}$

5. A bag contains 3 red, 6 white and 7 blue balls. 2 balls are drawn at random. Find the probability that one is while and one is blue.

   (a) $\dfrac{7}{20}$ (b) $\dfrac{8}{20}$

   (c) $\dfrac{9}{20}$ (d) $\dfrac{11}{20}$

6. What is the chance that the year 2000 contains 53 Sundays ?

   (a) $\dfrac{1}{7}$ (b) $\dfrac{2}{7}$

   (c) $\dfrac{2}{9}$ (d) $\dfrac{5}{7}$

7. A bag contains 5 green, 4 yellow and 3 white balls. 3 balls drawn at random. What is the probability that they are not of the same colour ?

   (a) $\dfrac{31}{44}$ (b) $\dfrac{35}{44}$

   (c) $\dfrac{41}{44}$ (d) $\dfrac{42}{44}$

8. The probability that a teacher will give one Surprise Test during any class meeting in a week is $\dfrac{1}{5}$. If a student is absent for 2 days, what is the probability that he will miss at least one test ?

   (a) $\dfrac{4}{15}$ (b) $\dfrac{1}{17}$

   (c) $\dfrac{9}{25}$ (d) $\dfrac{16}{25}$

9. In a box containing 100 bulbs 10 are defective. What is the probability that out of a sample of 5 taken from the box none is defective ?

   (a) $\dfrac{^{90}C_5}{^{100}C_5}$

   (b) $\dfrac{^{90}C_4}{^{100}C_5}$

   (c) $\dfrac{^{90}C_5}{^{101}C_5}$

   (d) $\dfrac{^{95}C_5}{^{100}C_5}$

10. In a box carrying one dozen oranges one third have become bad. If 3 oranges are taken out from the box at random, what is the probability that at least one orange out of the three oranges picked up is good ?

   (a) $\dfrac{53}{55}$ (b) $\dfrac{54}{55}$

   (c) $\dfrac{45}{55}$ (d) $\dfrac{51}{55}$

Pie-charts

For a pie chart, the central angle is 360° that represents 100% of the value.

Hence, we have following table :

360° = 100%	3.6° = 1%	18° = 5%
180° = 50%	36° = 10%	54° = 15%
90° = 25%	72° = 20%	108° = 30%
45° = 12.5%	144° = 40%	

Pie-chart

Directions (1 to 5) : The pie-chart provided below gives the distribution of land (in a village) under various food crops. Study the pie-chart carefully and answer the questions that follow.
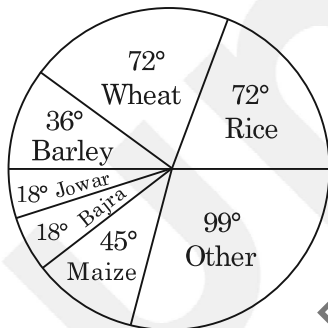
Distribution of areas (in acres) under various food crops.

1. Which combination of three crops contribute to 50% of the total area under the food crops ?
   (a) Wheat, Barley and Jowar
   (b) Rice, Wheat and Jowar
   (c) Rice, Wheat and Barley
   (d) Bajra, Maize and Rice

2. If the total area under Jowar was 1.5 million acres, then what the area (in million acres) under rice ?

   (a) 6   (b) 7.5
   (c) 9   (d) 4.5

3. If the production of wheat is 6 times that of barley, then what is the ratio between the yield per acre of wheat and barley ?
   (a) 3 : 2   (b) 3 : 1
   (c) 12 : 1   (d) 2 : 3

4. If the yield per acre of rice was 50% more than that of barley, then the production of barley is what percent of that of rice ?

   (a) 30%   (b) $33\frac{1}{3}\%$

   (c) 35%   (d) 36%

5. If the total area goes up by 5%, and the area under wheat production goes up by 12%, then what will be the angle for wheat in the new pie-chart ?
   (a) 62.4°   (b) 76.8°
   (c) 80.6°   (d) 84.2°

Directions (6 to 10) : Study the following graph carefully and answer the questions given below it. The pie-chart given below shows the spendings of a country on various sports during a particular year.
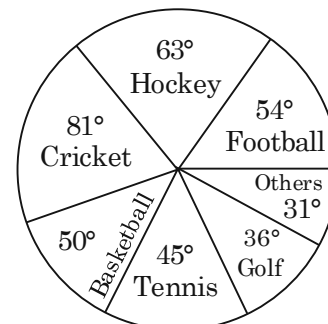
1. Consider the four age pyramids given below namely A, B, C and D representing four different countries.
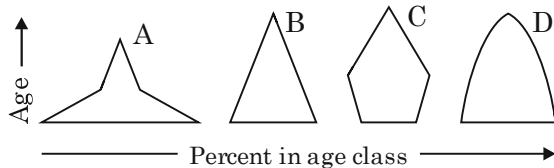
Which one of them indicates the declining population ?

(a) A (b) B

(c) C (d) D **[CSAT 2011]**

2. The following figure has four curves namely A, B, C and D. Study the figure and answer the item that follows.
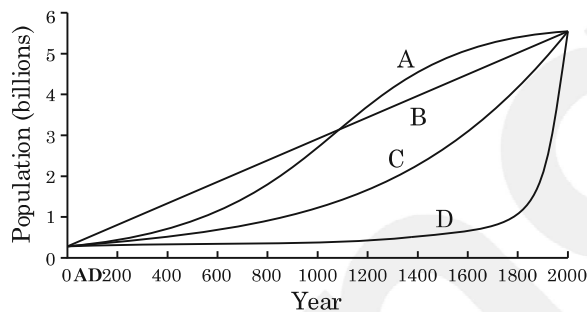
Which curve indicates the exponential growth?

(a) A (b) B

(c) C (d) D **[CSAT 2011]**

Directions for the following 2 (two) items :

The following pie-charts show the break-up of disease categories recorded in the patients from two towns, Town A and Town B. Pie-charts plot the disease categories as percentage of the total number of patients. Based on these, answer the two items that follow the charts.
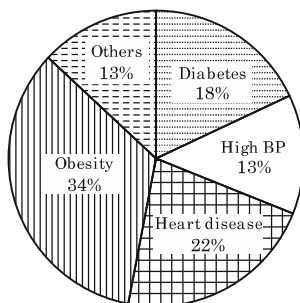
Distribution of diseases in Town-A
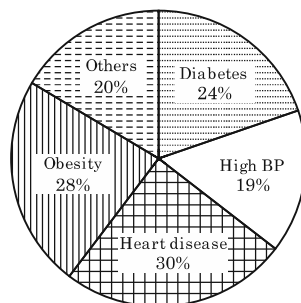
Distribution of diseases in Town-B

3. Which of the two towns has a higher number of persons with Diabetes ?

(a) Town A

(b) Town B

(c) Same in Town A and Town B

(d) No inference can be drawn **[CSAT 2011]**

4. What can we say about persons with more than one disease from these graphs ?

(a) There are likely to be persons with more than one disease in Town A.

(b) There are likely to be persons with more than one disease in Town B.

(c) There are likely to be persons with more than one disease in both Towns A and B.

(d) No inference can be drawn.

[CSAT 2011]

5. Consider the following Velocity-Time graph. It shows two trains starting simultaneously on parallel tracks.
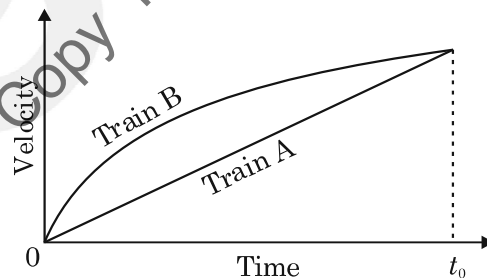
With reference to the above graph, which one of the following statements is *not* correct ?

(a) Train B has an initial acceleration greater than that of Train A.

(b) Train B is faster than Train A at all times.

(c) Both trains have the same velocity at time t_0.

(d) Both trains travel the same distance in time t_0 units.

[CSAT 2011]

Logical Reasoning

Chapter 16 — Number Series

Addition / Subtraction based Series:

1. Find the missing term in the given series
   2, 8, 14, 24, 34, 48, ?
   a. 66   b. 62
   c. 58   d. 64

2. 7, 10, 16, 28, ?, 100
   a. 48   b. 52
   c. 58   d. 64

3. Find the missing term in the given series
   9, 11, 16, 26, ?, 69
   a. 66   b. 62
   c. 58   d. 43

4. 21, 24, 32, 45, 63, ?
   a. 86   b. 76
   c. 85   d. 96

5. Find the missing term in the given series
   7, 10, 16, 28, 52, 100, ?
   a. 196   b. 144
   c. 151   d. 164

6. 7, 20, 46, 85, ?, 202
   a. 148   b. 137
   c. 158   d. 164

7. Find the missing term in the given series
   2, 3, 7, 16, 32, ?
   a. 57   b. 59
   c. 56   d. 60

8. 8, 14, 24, 38, 56, ?
   a. 55   b. 65
   c. 96   d. 78

9. Find the next term in the given series
   1, 2, 6, 24, 120, ….?
   a. 360   b. 420
   c. 720   d. 800

10. 240, …, 120, 40, 10, 2
    a. 480   b. 240
    c. 220   d. 120

Multiplication / Division based Series:

11. Find the missing term in the given series
    1, 1.5, 3, 4.5, 9, 13.5, ?
    a. 20   b. 25
    c. 27   d. 27.5

12. 3, 4.5, 9, 22.5, 67.5, ..., 945
    a. 265.25   b. 236.25
    c. 225.36   d. 150

13. Find the missing term in the given series
    4, 4, ..., 12, 30
    a. 5   b. 6
    c. 8   d. 10

14. 15, 30, 60, 120, 240, ?
    a. 240   b. 420
    c. 480   d. None

15. Find the missing term in the given series
    360, 180, 90, 45, 22.5, ?
    a. 20   b. 15.75
    c. 11.25   d. 11.5

16. 3, 6, 18, 72, ?
    a. 144   b. 216
    c. 288   d. 360

17. Find the next term in the given series
    1, 4, 9, 16, 25, ….?
    a. 36   b. 49
    c. 37   d. 40

18. 1, 8, 27, 64, 125, ….?
    a. 216   b. 250
    c. 175   d. 200

19. Find the missing term in the given series
    0, 10, 24, 68, 120, 222, ……
    a. 340   b. 250
    c. 350   d. 336

20. 2, 12, 36, 80, 150, 252, ……
    a. 144   b. 216
    c. 288   d. 392

24. 36, 20, 12, 8, 6, 5.5, 4.5
    a. 5.5   b. 6
    c. 12   d. 20

25. 2, 3, 10, 40, 172, 885, 5346
    a. 3   b. 855
    c. 40   d. 172

26. 10, 26, 74, 218, 654, 1946, 5834
    a. 26   b. 74
    c. 218   d. 654

27. 15, 16, 34, 105, 424, 2124, 12576
    a. 16   b. 34
    c. 105   d. 2124

28. 1, 3, 9, 31, 128, 651, 3913
    a. 651   b. 128
    c. 31   d. 9

29. 32, 36, 41, 61, 86, 122, 171, 235
    a. 41   b. 61
    c. 86   d. 122

30. 1, 2, 8, 33, 148, 760, 4626
    a. 2   b. 8
    c. 33   d. 760

31. 2807, 1400, 697, 347, 171, 84, 41, 20
    a. 697   b. 347
    c. 171   d. 84

32. 3, 8, 18, 46, 100, 210, 432
    a. 8   b. 18
    c. 46   d. 100

33. 5, 8, 20, 42, 124, 246, 736
    a. 8   b. 20
    c. 42   d. 124

34. 888, 440, 216, 104, 48, 22, 6
    a. 440   b. 216
    c. 104   d. 22

35. 4, 5, 15, 49, 201, 1011, 6073
    a. 5   b. 15
    c. 49   d. 201

36. 789, 645, 545, 481, 440, 429, 425
    a. 645   b. 545
    c. 481   d. 440

37. 5, 10, 17, 27, 37, 50, 65
    a. 10   b. 17
    c. 27   d. 37

38. 0, 1, 9, 36, 99, 225, 441
    a. 9   b. 36
    c. 99   d. 225

39. 2, 3, 11, 38, 102, 229, 443
    a. 11   b. 229
    c. 120   d. 38

40. 10, 15, 24, 35, 54, 75, 100
    a. 35   b. 75
    c. 24   d. 15

41. 3, 2, 3, 6, 12, 37.5, 115.5
    a. 37.5   b. 12
    c. 6   d. 2

42. 2, 8, 32, 148, 765, 4626, 32431
    a. 765   b. 148
    c. 8   d. 32

43. 1, 3, 4, 7, 11, 18, 27, 47
    a. 27   b. 11
    c. 18   d. 7

44. 2, 3, 5, 8, 14, 23, 41, 69
    a. 5   b. 8
    c. 69   d. 41

45. 3, 10, 35, 172, 885, 5346, 37471
    a. 10   b. 5346
    c. 885   d. 35

46. 318, 158, 76, 38, 18, 83
    a. 38   b. 18
    c. 158   d. 76

ANSWER KEY

1.	b	2.	a	3.	b	4.	c	5.	c	6.	a	7.	d	8.	c	9.	d	10.	a
11.	c	12.	c	13.	a	14.	d	15.	d	16.	b	17.	b	18.	b	19.	d	20.	b
21.	d	22.	d	23.	a	24.	a	25.	c	26.	d	27.	d	28.	b	29.	a	30.	d
31.	b	32.	b	33.	b	34.	d	35.	a	36.	d	37.	c	38.	c	39.	b	40.	a
41.	b	42.	d	43.	a	44.	c	45.	d	46.	d										

Previous Year Questions

01. What is the missing number 'X' of the series
7, X, 21, 31, 43 ?
(a) 11   (b) 12
(c) 13   (d) 14   [CSAT 2015]

02. What is X in the sequence
4, 196, 16, 144, 36, 100, 64, X ?
(a) 48   (b) 64
(c) 125   (d) 256   [CSAT 2019]

03. A simple mathematical operation in each number of the sequence 14, 18, 20, 24, 30, 32, ... results in a sequence with respect to prime number. Which one of the following is the next number in the sequence?
(a) 34   (b) 36
(c) 38   (d) 40   [CSAT 2020]

04. Replace the incorrect term by the correct term in the given sequence
3, 2, 7, 4, 13, 10, 21, 18, 31, 28, 43 ,40
where odd terms and even terms follow the same pattern.
(a) 0   (b) 1
(c) 3   (d) 6   [CSAT 2021]

05. You are given two identical sequences in two rows:

Sequence-I	8	4	6	15	52.5	236.25
Sequence-II	5	A	B	C	D	E

What is the entry in the place of C for the Sequence-II?
(a) 2.5   (b) 5
(c) 9.375   (d) 32.8125   [CSAT 2021]

06. What is the value of X in the sequence
20, 10, 10, 15, 30, 75, X?
(a) 105   (b) 120
(c) 150   (d) 225   [CSAT 2022]

07. What is the value of X in the sequence
2, 12, 36, 80, 150, X?
(a) 248   (b) 252
(c) 258   (d) 262   [CSAT 2022]
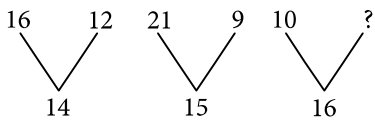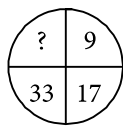
ANSWER KEY

01. (c)   02. (b)   03. (c)   04. (a)   05. (c)   06. (d)   07. (b)

In following question find missing number

1.

$$
\begin{array}{ccccccc}
16 \quad 12 & & 21 \quad 9 & & 10 \quad ? \\
\diagdown\!\!\!\diagup & & \diagdown\!\!\!\diagup & & \diagdown\!\!\!\diagup \\
14 & & 15 & & 16
\end{array}
$$

     a. −21   b. 12
     c. 32   d. 22

2.

?	9
33	17
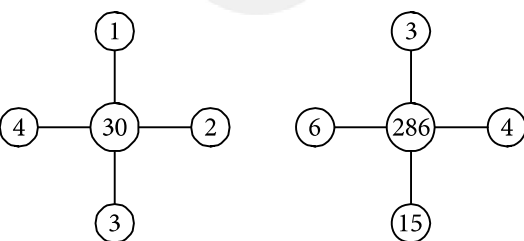
     a. 60   b. 68
     c. 55   d. 65

3.

169	64	81	30
625	?	49	50
1296	576	100	70

     a. 324   b. 289
     c. 441   d. 361

4.

3	4	5
2	3	4
1	2	3
14	29	?

     a. 50   b. 30
     c. 40   d. 32

5.

     a. 6   b. 7
     c. 9   d. 12

6.
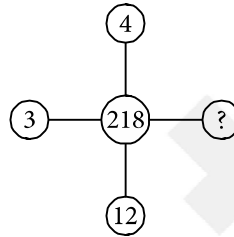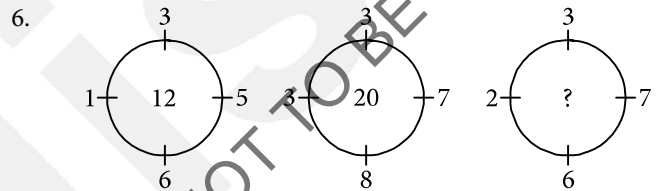
     a. 10   b. 15
     c. 20   d. 25

7.

5	9	15
16	26	?
49	89	147

     a. 45   b. 48
     c. 51   d. 54

8.

$$
\begin{pmatrix}
2 & 3 & 1 \\
1 & 2 & -1 \\
3 & 4 & ?
\end{pmatrix}
$$

     a. 5   b. 2
     c. 1   d. 4

9.

3	4	5
4	5	3
4	3	?
48	60	105

     a. 2   b. 6
     c. 7   d. 9

Previous Year Questions

01. Consider the following figures:

2	6
80	24

3	?
120	36

What is the missing number?
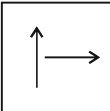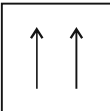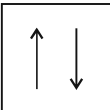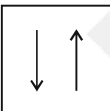
(a) 7 (b) 8

(c) 9 (d) 10 **[CSAT 2011]**

02. Consider the following matrix:

Which one of the following figures fits into the blank part of the above matrix?

(a)

(b)

(c)

(d)   **[CSAT 2014]**

03. Consider the table given below in which the numbers bear certain relationship among themselves along the rows:

29	13	18
33	X	19
30	27	3
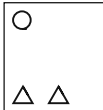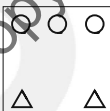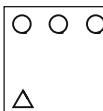
Which one of the following numbers is the missing number indicated above by X?

(a) 19 (b) 15

(c) 14 (d) 8 **[CSAT 2014]**

04. Consider the following matrix with one empty block in the lower extreme corner:

Which of the following figures could fit in the empty block and thus complete the matrix?

(a)   (b)

(c)   (d)

[CSAT 2014]

05. Consider the figure given below:

To fit the question mark, the correct answer is

(a)

(b)

(c)

(d)   **[CSAT 2015]**

Clocks

Key points:

- Dial of a clock is divided into 60 equal divisions which are known as minute spaces.

- A clock has two hands, the smaller one is called the hour hand and the larger one is called the minute hand (however, some clocks have second hand also but usually it is not taken into account since it moves all the time).

- There are 60 seconds in 1 minute and there are 60 minutes in 1 hour.

- Minute hand and hour hand coincide once in every 1 hour and 22 times in a day.

- Minute hand and hour hand are in opposite directions once in every 1 hour and 22 times in a day.

- Minute hand and hour hand subtend 90° twice in every 1 hour and 44 times in a day.

- Angle swiped by hour hand in 12 hours = 360° and in 1 hour = 30°.

- Angle swiped by minute hand in 60 minutes = 360° and in 1 minute = 6°.

- At 'n' O'clock, difference between hour hand and minute hand is 5n divisions.

- In 1 hour, minute hand overtakes hour hand by 55 divisions.

- Minute hand overtakes hour hand by 55 divisions in 60 minutes. Hence, 1 division in $\dfrac{12}{11}$ minutes.

Angle between hour hand and minute hand:

$\left| \dfrac{11\,M - 60\,H}{2} \right|^{\circ}$; where, M = minute and H = hour

Ex. 1 Find out the angle swiped by an hour hand from 12 noon to 5 : 30 pm.

Sol: Angle swiped by hour hand in 1 hour = 30°. So in 5.5 hours = 5.5 × 30° = 165°.

Ex. 2 Find the angle between hour and minute hands of the clock at 8:30?

Sol: Angle between the hour hand and the minute hand of a clock is given by

$\left| \dfrac{11\,M - 60\,H}{2} \right|^{\circ}$; where, M = 8 and H = 30.

Substituting the values, we get $\left| \dfrac{11 \times 30 - 60 \times 8}{2} \right|^{\circ} = \left| \dfrac{330 - 480}{2} \right|^{\circ} = \left| \dfrac{150}{2} \right|^{\circ} = 75°$

Ex. 3 At what time between 4 and 5 o'clock, hands of a clock are together.

Sol: At 4 o'clock, hands of a clock are 20 divisions apart. For being together minute hand will have to overtake hour hand by 20 divisions.

Minute hand overtakes hour hand by 55 divisions in 60 minutes, hence, 1 division in $\dfrac{12}{11}$ minutes.

So, 20 divisions in $= 20 \times \dfrac{12}{11}$ minutes $= \dfrac{240}{11}$ minutes

Previous Year Questions

01. Assume that
    1. the hour and minute hands of a clock move without jerking.
    2. the clock shows a time between 8 o'clock and 9 o'clock.
    3. the two hands of the clock are one above the other.

    After how many minutes (nearest integer) with the two hands will be again lying one above the other?

    (a) 60   (b) 62

    (c) 65   (d) 67   **[CSAT 2014]**

02. Between 6 PM and 7 PM the minute hand of a clock will be ahead of the hour hand by 3 minutes at

    (a) 6 : 15 PM

    (b) 6 : 18 PM

    (c) 6 : 36 PM

    (d) 6 : 48 PM   **[CSAT 2015]**

03. A watch loses 2 minutes in every 24 hours while another watch gains 2 minutes in every 24 hours. At a particular instant, the two watches showed an identical time. Which of the following statements is correct if 24 hour clock is followed?

    (a) The two watches show the identical time again on completion of 30 days.

    (b) The two watches show the identical time again on completion of 90 days.

    (c) The two watches show the identical time again on completion of 120 days.

    (d) None of the above statements is correct.

             [CSAT 2017]

04. A clock strikes once at 1 o'clock, twice at 2 o'clock and thrice at 3 o'clock, and so on. If it takes 12 seconds to strike at 5 o'clock, what is the time taken by it to strike at 10 o'clock?

    (a) 20 seconds

    (b) 24 seconds

    (c) 28 seconds

    (d) 30 seconds   **[CSAT 2017]**

05. A wall clock moves 10 minutes fast in every 24 hours. The clock was set right to show the correct time at 8:00 a.m. on Monday. When the clock shows the time 6:00 p.m. on Wednesday, what is the correct time?

    (a) 5:36 pm   (b) 5:30 pm

    (c) 5:24 pm   (d) 5:18 pm   **[CSAT 2019]**

06. At which one of the following times, do the hour hand and the minute hand of the clock make an angle of 180° with each other?

    (a) At 7:00 hours.   **[CSAT 2021]**

    (b) Between 7:00 hours and 7:05 hours.

    (c) At 7:05 hours.

    (d) Between 7:05 hours and 7:10 hours.

07. Consider the following statements :
    1. Between 3:16 p.m. and 3:17 p.m., both hour hand and minute hand coincide.
    2. Between 4:58 p.m. and 4:59 p.m., Both minute hand and second hand coincide.

    Which of the above statements is/are correct?

    (a) 1 only   (b) 2 only

    (c) Both 1 and 2   (d) Neither 1 nor 2

             [CSAT 2022]

ANSWER KEY

01. (c)   02. (c)   03. (d)   04. (b)   05. (a)   06. (d)   07. (c)

Chapter **19** Calendar

Odd days

In a given period, the number of days more than the complete weeks are called *odd days*. Odd days are always between 0-6 days. If odd days are more than 7, then divide it by '7', remainder will be considered as number of odd days.

For 100 years = 76 ordinary years + 24 leap years = $(76 \times 1 + 24 \times 2)$ odd days = 124 odd days

$$= (17 \text{ weeks} + 5 \text{ days}) = 5 \text{ odd days}.$$

Number of odd days in 100 years = 5

Number of odd days in 200 years = $(5 \times 2) \equiv 3$ odd days.

Number of odd days in 300 years = $(5 \times 3) \equiv 1$ odd day.

Number of odd days in 400 years = $(5 \times 4 + 1) \equiv 0$ odd day.

Similarly, each one of 800 years, 1200 years, 1600 years, 2000 years etc. has 0 odd day.

Normal year

1. Contains 365 days.
2. Contains 52 weeks and 1 odd day (since $52 \times 7 = 364$, hence, 365th day is considered as odd day).
3. An ordinary year starts and ends with the same day of the week.
4. First day of the year occurs '53' times and rest days occur '52' times in the year.

Leap year

1. Contains 366 days.
2. Contains 52 weeks and 2 odd days (since $52 \times 7 = 364$, hence 365th and 366th day are considered as odd days).
3. A leap year ends with the next day of the first day of the year.
4. First 2 days of the year occur '53' times and rest '52' times.
5. If a century year is a multiple of 400, then only it is a leap year else not and if a non-century year is a multiple of 4, then it is a leap year.

Key points:

1. For any leap year the calendar of January and July is exactly same and January and April is same till 30th.
2. For any year the calendar of March and November is same till 30th.
3. For a normal year calendar of January is same as October.
4. Calendar of a year which is just after a leap year repeat after every 6 years, and that of a year which is just before a leap year, repeats after every 11 years provided and the one before it the non-leap century.

20. A certain day, which is x days before 17th August, is such that 50 days prior to that day, it was 4x days since March 30th of the same year. What is x ?
    a. 18   b. 30
    c. 22   d. 16

21. Ketan takes casual leave only on first working day of every month. The office has weekly offs on Saturday and Sunday. In a month of 30 days, the first working days happened to be Tuesday. What will be the day for his next casual leave ?
    a. Wednesday   b. Thursday
    c. Friday   d. Monday

22. Abhay gave an application for a new ration card to the clerk on Monday afternoon. Next day was a holiday. So the clerk cleared the papers on the next working day on resumption of duty. The senior clerk checked it on the same day but forwarded it to the head clerk on next day. The head clerk decided to dispose the case on the subsequent day. On which of the following days was the case put up to the head clerk by the senior clerk ?
    a. Wednesday   b. Thursday
    c. Friday   d. Saturday

ANSWER KEY

1. b 2. c 3. d 4. b 5. d 6. c 7. d 8. a 9. b 10. c
11. a 12. b 13. c 14. c 15. a 16. c 17. a 18. a 19. d 20. a
21. b 22. b

Previous Year Questions

01. If the 3rd day of a month is Monday, which one of the following will be the fifth day from 21st of this month?

    (a) Monday
    (b) Tuesday
    (c) Wednesday
    (d) Friday   **[CSAT 2014]**

02. If second and fourth Saturdays and all the Sundays are taken as only holidays for an office, what would be the minimum number of possible working days of any month of any year?

    (a) 23
    (b) 22
    (c) 21
    (d) 20   **[CSAT 2017]**

03. Consider the sequence given below:

    4/12/95, 1/1/96, 29/1/96, 26/2/96, …

    What is the next term of the series?

    (a) 24/3/96
    (b) 25/3/96
    (c) 26/3/96
    (d) 27/3/96   **[CSAT 2018]**

04. Mr 'X' has three children. The birthday of the first child falls on the 5th Monday of April, that of the second one falls on the 5th Thursday of November. On which day is the birthday of his third child, which falls on 20th December?

    (a) Monday
    (b) Thursday
    (c) Saturday
    (d) Sunday   **[CSAT 2019]**

05. Which year has the same calendar as that of 2009?

    (a) 2018
    (b) 2017
    (c) 2016
    (d) 2015   **[CSAT 2019]**

06. If in a particular year 12th January is a Sunday, then which one of the following is correct?

    (a) 15th July is a Sunday if the year is a leap year.
    (b) 15th July is a Sunday is the year is not a leap year.
    (c) 12th July is a Sunday if the year is a leap year.
    (d) 12th July is not a Sunday if the year is a leap year.

    [CSAT 2020]

07. Consider two Statements and a Question:

    Statement-1: The last day of the month is a Wednesday.
    Statement-2: The third Saturday of the month was the seventeenth day.
    Question: What day is the fourteenth of the given month?

    (a) Statement-1 alone is sufficient to answer the Question.
    (b) Statement-2 alone is sufficient to answer the Question.
    (c) Both Statement-1 and Statement-2 are required to answer the Question.
    (d) Neither Statemetn-1 alone nor Statement-2 alone is sufficient to answer the Question.

    [CSAT 2021]

08. Which day is 10th October, 2027?

    (a) Sunday
    (b) Monday
    (c) Tuesday
    (d) Saturday   **[CSAT 2021]**

09. Joseph visits the club on every 5th day, Harsh visits on every 24th day, while Sumit visits on every 9th day. If all three of them met at the club on a Sunday, then on which day will all three of them meet again?

    (a) Monday
    (b) Wednesday
    (c) Thursday
    (d) Sunday   **[CSAT 2021]**

10. Which date of June 2099 among the following is Sunday?

    (a) 4
    (b) 5
    (c) 6
    (d) 7   **[CSAT 2022]**

ANSWER KEY

| 01. (c)	02. (b)	03. (b)	04. (b)	05. (d)	06. (c)	07. (b)	08. (a)	09. (b)	10. (d)

1. If the total number of data on opposite faces of a cubical block is always 7, find the figure which is correct.

   a.   b.

   c.   d.
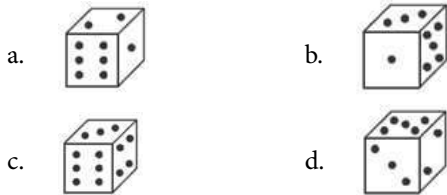
2. Amongst the following figures, find the correct one, if it is known that the total number of dots on opposite faces of the cube shown is always 7.

   a.   b.

   c.   d.
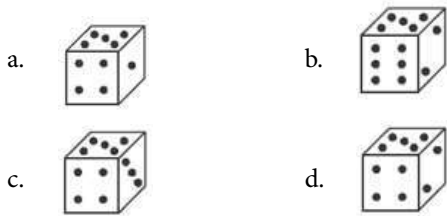
3. The four difference positions of a dice are given below:
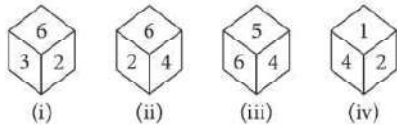
   Which number is on the face opposite 6?
   a. 1   b. 2
   c. 3   d. 4

4. How many dots are there on the dice face opposite the one with three dots?
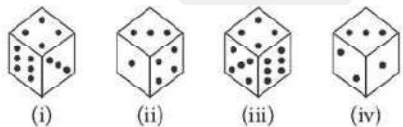
   a. 2
   b. 4
   c. 5
   d. 6

5. Observe the dots on a dice (one to six dots) in the following figures. How many dots are contained on the face opposite to that containing four dots?
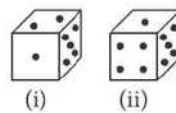
   a. 2
   b. 3
   c. 6
   d. Cannot be determined

6. Two positions of a dice are shown below. Identify the number at the bottom when the top is 3?
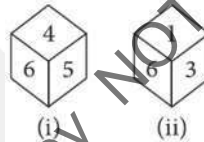
   a. 2   b. 4
   c. 5   d. 6

7. Two positions of a dice a are shown. When 4 is at the bottom, what number will be on the top?
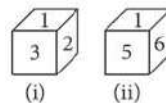
   a. 1   b. 2
   c. 5   d. 6

8. Two positions of a cube are shown below. When the number 4 will be at the bottom, then which number will be at the top?
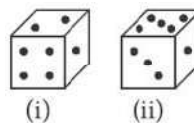
   a. 3
   b. 5
   c. 6
   d. Cannot be determined

Practice Set

1. The four different positions of a dice are given below:

   (i)   (ii)   (iii)   (iv)
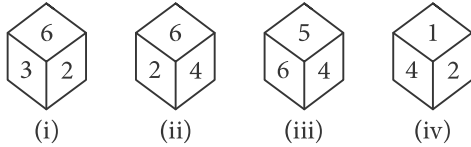
   Which number is on the face opposite 6?
   a. 1   b. 2
   c. 3   d. 4

2. How many dots are there on the dice face opposite the one with three dots?

   (i)   (ii)   (iii)   (iv)
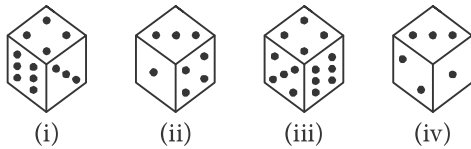
   a. 2   b. 4
   c. 5   d. 6

3. What is opposite 3, if four different positions of a dice are as shown below?

   (i)   (ii)   (iii)   (iv)
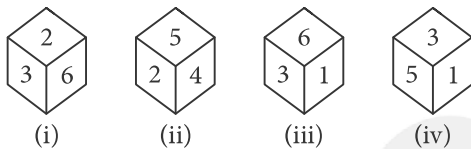
   a. 6   b. 4
   c. 3   d. 2

4. Show below are four different positions of the same dice. Find the number on the face opposite the face showing 6.

   (i)   (ii)   (iii)   (iv)
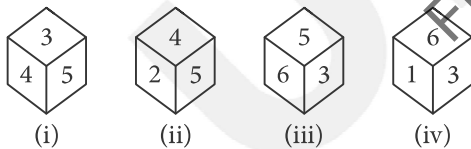
   a. 1   b. 2
   c. 4   d. 5

5. Four usual dice are thrown on the ground. The total of numbers on the top faces of these four dice is 13 as the top faces showed 4, 3, 1 and 5 respectively. What is the total of the faces touching the ground?
   a. 12   b. 13
   c. 15   d. Cannot be determined

6. Which number is on the face opposite 4? If the four different positions of a dice are as shown in the figures given below.

   (i)   (ii)   (iii)   (iv)
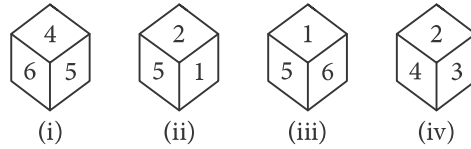
   a. 5   b. 3
   c. 2   d. 1

7. A dice is thrown four times and its four different positions are shown below. Find the number on the face opposite the face showing 2.

   (i)   (ii)   (iii)   (iv)
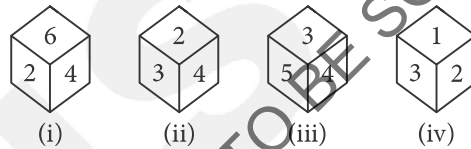
   a. 3   b. 4
   c. 5   d. 6

8. Four positions of a dice are shown below. What number must be at the bottom face when the dice is in the position as shown in figure (iii)?

   (i)   (ii)   (iii)   (iv)
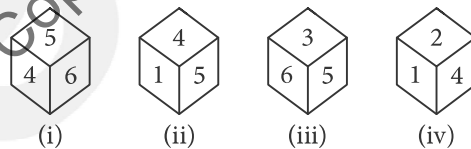
   a. 1   b. 2
   c. 4   d. 6

9. If the total number of data on opposite faces of a cubical block is always 7, find the figure which is correct.

   a.

   b.

   c.

   d.
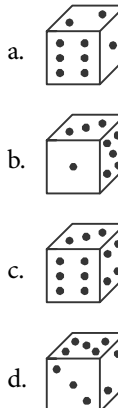
Previous Year Questions

01. Six squares are coloured, front and back, red (R), blue (B), yellow (Y), green (G), white (W) orange (O) and are hinged together as shown in the figure below. If they were folded to form a cube, what would be the face opposite to white face?
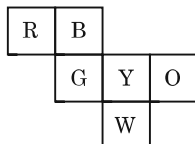
(a) R (b) G
(c) B (d) O **[CSAT 2012]**

02. The views of a cube following a particular motion are given below:
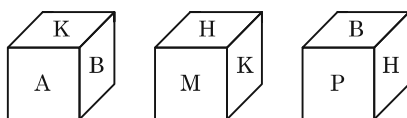
What is the letter opposite to A?
(a) H (b) P
(c) B (d) M **[CSAT 2012]**

03. A cube has six numbers marked 1, 2, 3, 4, 5 and 6 on its faces. Three views of the cube are shown below:
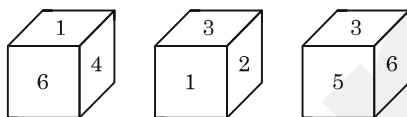
What possible number can exist on the two faces marked Ⓐ and Ⓑ, respectively on the cube?
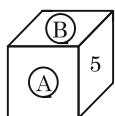
(a) 2 and 3 (b) 6 and 1
(c) 1 and 4 (d) 3 and 1 **[CSAT 2013]**

04. Each of the six different faces of a cube has been coated with a different colour i.e., V, I, B, G, Y and O. Following information is given:
1. Colours Y, O and B are on adjacent faces.
2. Colours I, G and Y are on adjacent faces.
3. Colours B, G and Y are on adjacent faces

Which is the colour of the face opposite to the face coloured with O?
(a) B (b) V
(c) G (d) I **[CSAT 2015]**

05. A cube has all its faces painted with different colours. It is cut into smaller cubes of equal sizes such that the side of the small cube is one-fourth the big cube. The number of small cubes with only one of the sides painted is
(a) 32 (b) 24
(c) 16 (d) 8 **[CSAT 2016]**

06. The outer surface of a 4 cm × 4 cm × 4 cm cube is painted completely in red. It is sliced parallel to the faces to yield sixty four 1 cm × 1 cm × 1 cm small cubes. How many small cubes do not have painted faces?
(a) 8 (b) 16
(c) 24 (d) 36 **[CSAT 2017]**

Directions for the following 3 (three) items:
Rotated positions of a single solid are shown below. The various faces of the solid are marked with different symbols like dots, cross and line. Answer the three items that follow the given figures.
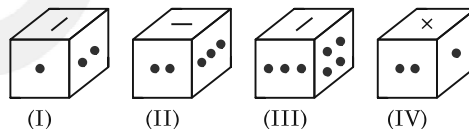
07. What is the symbol on the face opposite to that containing a single dot?
(a) Four dots (b) Three dots
(c) Two dots (d) Cross **[CSAT 2018]**

08. What is the symbol on the face opposite to that containing two dots?
(a) Single dot (b) Three dots
(c) Four dots (d) Line **[CSAT 2018]**

09. What is the symbol on the face opposite to that containing the cross?
(a) Single dot (b) Two dots
(c) Line (d) Four dots **[CSAT 2018]**

Blood relation

1. Pointing to a photograph X said to Y, "she is the only daughter of the father of my mother". How X is related to the person of the photograph?
   - (a) Daughter
   - (b) Nephew
   - (c) Son
   - (d) Son or Daughter

2. Pointing towards a girl Abhishek says, "This girl is the daughter of only child of my father." What is the relation of Abhishek's wife to that girl?
   - (a) Daughter
   - (b) Sister
   - (c) Aunt
   - (d) Mother

3. A is brother of B; B is brother of C; D is the father of A. On the basis of the given information, which one of the following is not definitely true?
   - (a) A is brother of C
   - (b) C is brother of A
   - (c) A, B, C are children of D
   - (d) B is the son of D

4. Pointing to a person, Deepak said, "His only brother is the father of my daughter's father". How is the person related to Deepak?
   - (a) Father
   - (b) Grandfather
   - (c) Uncle
   - (d) Brother in law

5. If A is brother of B; B is sister of C; C is father of D, how D is related to A?
   - (a) Niece
   - (b) Sister
   - (c) Nephew
   - (d) Cannot be determined

6. If D is Brother of B, how B is related to C? To answer this question which of the statements is/are necessary?
   1. The son of D is the grandson of C.
   2. B is the sister of D.
   - (a) Only 1
   - (b) Only 2
   - (c) Either 1 or 2
   - (d) 1 and 2 both are necessary

7. The statement: "The father of my son is the only child of your parents"
   - (a) can never be true.
   - (b) is true in only one type of relation.
   - (c) can be true for more than one type of relations.
   - (d) can be true only in a polygamous family.

8. If Sangeeta's daughter is my daughter's mother, then how am I related to Sangeeta?
   - (a) Son is the only possibility.
   - (b) Son-in-law is the only possibility.
   - (c) Daughter is the only possibility.
   - (d) Son-in-law or daughter.

Chapter 22 — Direction sense

1. If someone goes north, turn right, then turn right again and then goes to left. In which direction is the person now?

2. A, B, C and D are playing cards. A and B are partners. D faces towards North. If A faces towards the West, then in which direction C is facing (assume that partners are facing each other)?

3. If at 4:30 pm, the minute hand of a clock points towards East, in which direction the hour hand points?

4. Hari walks a distance of 3 km towards North, then turns to his left and walks for 2 km, he again turns left and walks for 3 km. At this point, he turns to his left and walks for 3 km. How many kilometers is he from the starting point?

5. Pradeep goes 10 km west, then turns left and goes 16 km, then turns right and goes 10 km and again turns right and goes 16 km. At what distance is he from the starting point now?

6. Priyanka walks 10 km towards North, from there she walks 6 km towards the South, then, she walks 3 km towards the East. How far and in which direction is she with reference to her starting point?

7. One morning after sunrise, Vimal started to walk. During this walk he met Ajay who was coming from opposite direction. Vimal observes the shadow of Ajay to the right of him (Vimal). To which direction Vimal was facing?

8. One morning after sunrise Nivedita and Niharika were talking to each other face to face. If Niharika's shadow was exactly to the right of Nivedita, which direction Niharika was facing?

01. The houses of A and B face each other on a road going north-south, A's being on the western side. A comes out of his house, turns left, travels 5 km, turns right, travels 5 km to the front of D's house. B does exactly the same and reaches the front of C's house. In this context, which one of the following statements is *correct*?

    (a) C and D live on the same street.

    (b) C's house faces south.

    (c) The houses of C and D are less than 20 km apart.

    (d) None of the above   **[CSAT 2011]**

02. Consider the following statements:

    There are six villages A, B, C, D, E and F.

    F is 1 km to the west of D.

    B is 1 km to the east of E.

    A is 2 km to the north of E.

    C is 1 km to the east of A.

    D is 1 km to the south of A.

    Which three villages are in a line?

    (a) A, C, B   (b) A, D, E

    (c) C, B, F   (d) E, B, D   **[CSAT 2014]**

03. Location of B is north of A and location of C is east of A. The distances AB and AC are 5 km and 12 km respectively. The shortest distance (in km) between the locations B and C is

    (a) 60   (b) 13

    (c) 17   (d) 7   **[CSAT 2014]**

04. Shahid and Rohit start from the same point in opposite directions. After each 1 km, Shahid always turns left and Rohit always turns right. Which of the following statements is correct?

    (a) After both have travelled 2 km, the distance between them is 4 km.

    (b) They meet after each has travelled 3 km.

    (c) They meet for the first time after each has travelled 4 km.

    (d) They go on without ever meeting again.

                                        [CSAT 2015]

05. A person X was driving in a place where all roads ran either north-south or east-west, forming a grid. Roads are at a distance of 1 km from each other in a parallel. He started at the intersection of two roads, drove 3 km north, 3 west and 4 km south. Which further route could bring him back to his starting point, if the same route is not repeated?

    (a) 3 km east, then 2 km south

    (b) 3 km east, then 1 km north

    (c) 1 km north, then 2 km west

    (d) 3 km south, then 1 km north   **[CSAT 2016]**

06. A person walks 12 km due north, then 15 km due east, after that 19 km due west and then 15 km due south. How far is the from the starting point?

    (a) 5 km   (b) 9 km

    (c) 37 km   (d) 61 km   **[CSAT 2016]**

07. P, Q and R are three towns. The distance between P and Q is 60 km, whereas the distance between P and R is 80 km. Q is in the West of P and R is in the South of P. What is the distance between Q and R?

    (a) 140 km   (b) 130 km

    (c) 110 km   (d) 100 km   **[CSAT 2019]**

08. 'A' started from his house and walked 20 m towards East, where his friend 'B' joined him. They together walked 10 m in the same direction. Then 'A' turned left while 'B' turned right and travelled 2 m and 8 m respectively. Again 'B' turned left to travel 4 m followed by 5 m to his right to reach his office. 'A' turned right and travelled 12 m to reach his office. What is the shortest distance between the two offices?

    (a) 15 m   (b) 17 m

    (c) 19 m   (d) 20 m   **[CSAT 2019]**

09. A man walks down the backside of his house straight 25 metres, then turns to the right and walks 50 metres again; then he turns towards left and again walks 25 metres. If his house faces to the East, what is his direction from the starting point?

    (a) South-East   (b) South-West

    (c) North-East   (d) North-West

                                        [CSAT 2020]

Chapter 23 — Coding Decoding

1. If, in a code, MIND becomes KGLB and ARGUE becomes YPESC, then what will DIAGRAM be in that code?
   - a. BGYEPYK
   - b. BGYPYEK
   - c. GLPEYKB
   - d. LKBGYPK

2. If GOLD is coded as HOME, COME is coded as DONE and CORD is coded as DOSE, how would you code SONS?
   - a. TPOT
   - b. TOOT
   - c. TOOS
   - d. TONT

3. In a certain code, kaveri is written as vakire. How is mysore written in that code?
   - a. EROSYM
   - b. SYMROE
   - c. SYMEOR
   - d. None of these

4. In a certain code language, GERMINATION is written as IMGRENNOAIT. How is ESTABLISHED written in that code?
   - a. BEATSLADEIHS
   - b. BAETSLEDISH
   - c. BATESLADEIHS
   - d. BAETSLDEIHS

5. In a certain code language, BOARD is written as EQBNC. How eill the word CLIMB be written in that language?
   - a. CLJKH
   - b. DKJLF
   - c. DNHMB
   - d. FNJRO

6. In a certain code, the words COME AT ONCE were written as XLNVZGLMXV. In the same code, which of the following could code OK?
   - a. KL
   - b. LM
   - c. KM
   - d. LP

7. If EHFNRQ is the code for BECKON, which word has the code QDFWXULQ?
   - a. NCQUTIRN
   - b. NACUTIRN
   - c. NATCRIUN
   - d. NACTURIN

8. If in a certain language, TRIANGLE is coded as SQHZMFKD, which word would be coded as DWZLOKD?
   - a. EXAMPLE
   - b. FIGMENT
   - c. DISMISS
   - d. DISJOIN

9. If fulfnhw is the code for cricket, then eulgh is the code for which word?
   - a. PRIDE
   - b. BRIDE
   - c. BLADE
   - d. BLIND

10. If in a certain language, itnietam is the code for intimate, which word has the code trevnietarbi?
    - a. INVRETIBRATE
    - b. INVERTIBARTE
    - c. INVERTIBRETA
    - d. INVERTIBRATE

11. If in a certain language, MACHINE is coded as LBBIHOD, which word would be coded as SLTMFNB?
    - a. RKSLEMA
    - b. RKULGMC
    - c. RMSNEOA
    - d. TMUNGOC

12. If deer = 12215 and high = 5645, how will you code heel?
    - a. 2328
    - b. 3449
    - c. 4337
    - d. 5229

13. If B is coded as 8, F is coded as 6, Q is coded as 4, D is coded as 7, T is coded as 2, M is coded as 3 and K is coded as 5, then what is the coded from of QKTBFM?
    - a. 425783
    - b. 452683
    - c. 452783
    - d. None of these

14. In a certain code, BRAIN is written as *%+#× and TIER is written as $#+%. How is RENT written in that code?
    - a. %×#$
    - b. %#×$
    - c. %+×$
    - d. +×%$

15. In a certain code, DEAF is written as 3587 and FILE is written as 7465. How is IDEAL written in that code?
    - a. 43568
    - b. 43586
    - c. 63548
    - d. 48536

16. If SCOTLAND is written as 12345678, LOAN is written as 1435, LOTS is written as 8124, DAN is written as 537 and SON is written as 458, then what will be the code for 'C'?
    - a. 0
    - b. 4
    - c. 5
    - d. 6

Previous Year Questions

01. A military code writes SYSTEM as SYSMET and NEARER as AENRER. Using the same code, FRACTION can be written as
    (a) CARFTION (b) FRACNOIT
    (c) NOITCARF (d) CARFNOIT
    [CSAT 2016]

02. In a certain code, '256' means 'red colour chalk', '589' means 'green colour flower' and '254' means 'white colour chalk'. The digit in the code that indicates 'white' is
    (a) 2 (b) 4
    (c) 5 (d) 8 **[CSAT 2017]**

03. If LSJXVC is the code for MUMBAI, the code for DELHI is
    (a) CCIDD (b) CDKGH
    (c) CCJFG (d) CCIFE **[CSAT 2018]**

04. If RAMON is written as 12345 and DINESH as 675849, then HAMAM will be written as
    (a) 92233 (b) 92323
    (c) 93322 (d) 93232 **[CSAT 2018]**

05. If every alternative letter of the English alphabet from B onwards (including B) is written in lower case (small letters) and the remaining letters are capitalized, then how is the first month of the second half of the year written?
    (a) JuLY (b) jULy
    (c) jUly (d) jUlY **[CSAT 2019]**

06. In the English alphabet, the first 4 letters are written in opposite order; and the next 4 letters are written in opposite order and so on; and at the end Y and Z are interchanged. Which will be the fourth letter to the right of the 13th letter?
    (a) N (b) T
    (c) H (d) I **[CSAT 2021]**

07. In a code language 'MATHEMATICS' is as 'LBSIDNZUHDR'. How is 'CHEMISTRY' written in that code language?
    (a) DIDLHRSSX (b) BIDNHTSSX
    (c) BIDLHTSSX (d) DGFLIRUQZ
    [CSAT 2021]

08. If the order of the letters in the English alphabet is reversed and each letter represents the letter whose position it occupies, then which one of the following represents 'LUCKNOW'?
    (a) OGXPMLD (b) OGXQMLE
    (c) OFXPMLE (d) OFXPMLD
    [CSAT 2022]

ANSWER KEY

01. (d) 02. (b) 03. (a) 04. (b) 05. (d) 06. (b) 07. (b) 08. (d)

1. Sarita is on 11th place from top in a group of 45 girls. If we start counting from bottom, what will be her place?
   (a) 36th
   (b) 34th
   (c) 35th
   (d) Cannot be determined

2. Raman is 9th from the top in a class of 31 students. What will be his position from the bottom?
   (a) 21st (b) 22nd
   (c) 23rd (d) 24th

3. Ram and Sham are ranked 13th and 14th respectively is a class of 23. What are their ranks from the last respectively?
   (a) 10th and 11th (b) 11th and 12th
   (c) 11th and 10th (d) None of these

4. If Nikhil is eleventh from the left in a row of boys, Rehaman is fourteenth from the right, how many boys are there in the row?
   (a) 25   (b) 23
   (c) 36   (d) Data inadequate

5. Ravi is 7 ranks ahead of Sumit in a class of 39. If Sumit's rank is seventeenth from the last, what is Ravi's rank from the start?
   (a) 14th (b) 15th
   (c) 16th (d) 17th

6. In a row at a bus stop, A is 7th from the left and B is 9th from the right. Both of them interchange their positions and thus A becomes 11th from the left. How many people are there in that row?
   (a) 19   (b) 20
   (c) 15   (d) 18

7. Some boys are sitting in a line. Mahendra is on 17th place from left and Surendra is on 18th place from right. There are 8 boys in between them. How many boys are there in the line?
   (a) 43   (b) 42
   (c) 41   (d) 44

8. In a row of students, Ramesh is 9th from the left and Suman is 6th from the right. When they both interchange their position then Ramesh will be 15th from the left. What will be the position of Suman from the right?
   (a) 12th (b) 13th
   (c) 15th (d) 6th

9. In a line of boys, Ganesh is 12th from the left and Rajan is 15th from the right. They interchange their positions. Now, Rajan is 20th from the right. What is the total number of boys in the class?
   (a) 30   (b) 29
   (c) 32   (d) 31

10. Rajnish is older than Rajesh and Raman. Ramesh is older than Rajesh but younger than Rajeev. Raman is older than Rajeev. Who among them is oldest?
    (a) Rajeev   (b) Rajesh
    (c) Rajnish   (d) Ramesh

Previous Year Questions

01. In a queue, Mr. X is fourteenth from the front and Mr. Y is seventeenth from the end, while Mr. Z is exactly in between Mr. X and Mr. Y. If Mr. X is ahead of Mr. Y and there are 48 persons in the queue, how many persons are there between Mr. X and Mr. Z?

    (a) 6   (b) 7

    (c) 8   (d) 9   **[CSAT 2011]**

02. In a class of 45 students, a boy is ranked 20th. When two boys joined, his rank was dropped by one. What is his new rank from the end?

    (a) 25th (b) 26th

    (c) 27th (d) 28th **[CSAT 2013]**

03. In a row 'A' is the 11th position from the left and 'B' is in the 10th position from the right. If 'A' and 'B' interchange, then 'A' becomes 18th from the left. How many persons are there in the row other than 'A' and 'B'?

    (a) 27   (b) 26

    (c) 25   (d) 24   **[CSAT 2014]**

04. In a class of 60 students, where the number of girls is twice that of boys, Kamal, a boy, ranked seventeenth from the top. If there are 9 girls ahead of Kamal, the number of boys in rank after him is

    (a) 13   (b) 12

    (c) 7   (d) 3   **[CSAT 2016]**

05. 15 students failed in a class of 52. After removing the names of failed students, a merit order list has been prepared in which the position of Ramesh is 22nd from the top. What is his position from the bottom?

    (a) 18th (b) 17th

    (c) 16th (d) 15th **[CSAT 2017]**

06. A is 16th from the left end in a row of boys and V is 18th from the right end. G is 11th from A towards the right and 3rd from V towards the right ends. How many boys are there in the row?

    (a) 40   (b) 41   **[CSAT 2020]**

    (c) 42   (d) Insufficient data

07. Consider two Statements and a Question:

    Statement-1: Priya is 4 ranks below Seema and is 31st from the bottom.

    Statement-2: Ena is 2 ranks above Seema and is 37th from the bottom.

    Question: What is Seema's rank from the top in the class of 40 students?

    Which one of the following is correct in respect of the Statements and the Question?

    (a) Statement-1 alone is not sufficient to answer the Question.

    (b) Statement-2 alone is not sufficient to answer the Question.

    (c) Either Statement-1 alone or Statement-2 alone is sufficient to answer the Question.

    (d) Both Statement-1 and Statement-2 are required to answer the Question.

    [CSAT 2021]

08. Three persons A, B and c are standing in a queue not necessarily in the same order. There are 4 person between A and B, and 7 persons between B and C. If there are 11 persons ahead of C and 13 behind A, what could be the minimum number of persons in the queue?

    (a) 22

    (b) 28

    (c) 32

    (d) 38   **[CSAT 2022]**

ANSWER KEY

01. (c)   02. (c)   03. (c)   04. (b)   05. (c)   06. (b)   07. (c)   08. (a)

Chapter 25 — Mathematical operation

1. If '+' means 'minus' '–' means 'multiplied by' '÷' means 'Plus' and '×' means 'divided by', then $10 \times 5 \div 3 - 2 + 3 = ?$
   a. 5   b. 21
   c. 53/3   d. 18

2. Which one of the following is correct ?
   6 * 4 * 9 * 15
   a. ×, =, –   b. ×, –, =
   c. =, ×, –   d. –, ×, =

3. If '–' stands for division, '+' for multiplication '÷' for subtraction and '×' for addition. Which one of the following equation is correct ?
   a. $6 \div 20 \times 12 + 7 - 1 = 70$
   b. $6 + 20 - 20 \div 7 \times 1 = 62$
   c. $6 - 20 \div 12 \times 7 + 1 = 57$
   d. $6 + 20 - 20 \div 7 - 1 = 38$

4. If L stands for +, M stands for –, N stands for ×, P stands for ÷, then 14 N 10 L 42 P 2 M 8 = ?
   a. 153   b. 216
   c. 248   d. 251

5. If P denotes +, Q denotes –, R denotes × and S denotes ÷, which of the following statements is correct ?
   a. 36 R 4 S 8 Q 7 P 4 = 10
   b. 16 R 12 P 49 S 7 Q 9 = 200
   c. 32 S 8 R 9 = 160 Q 12 R 12
   d. 8 R 8 P 8 S 8 Q 8 = 57

Directions for questions (6-7): In an imaginary language, the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 are substituted by a, b, c, d, e, f, g, h, i and j. And 10 is written as ba.

6. (cd + ef) × bc is equal to
   a. 684   b. 816
   c. 916   d. 1564

7. baf + fg – (ca × h/be) is equal to
   a. 141
   b. 145
   c. 151
   d. 161

8. If '+' means '×', '×' means '–', '÷' means '+' and '–' means '÷', then which of the following gives the result of
   $175 - 25 \div 5 + 20 \times 3 \div 10 = ?$
   a. 114   b. 160
   c. 240   d. 2370

9. If the given interchanges namely : signs + and ÷ and numbers 2 and 4 are made in signs and numbers, which one of the following four equations would be correct ?
   a. $2 + 1 \div 3 = 3$ b. $4 + 2 \div 6 = 1.5$
   c. $4 \div 2 + 3 = 4$ d. $2 + 4 \div 6 = 8$

10. If X stands for 'addition' , < for 'subtraction', + stands for 'division', > for 'multiplication', – stands for 'equal to', ÷ for 'greater than' and = stands for 'less than', state which of the following is true ?
   a. $3 \times 2 < 4 + 16 > 2 + 4$
   b. $5 > 2 + 2 = 10 < 4 \times 2$
   c. $3 \times 4 > 2 - 9 + 3 < 3$
   d. $5 \times 3 < 7 \div 8 + 4 \times 1$

11. If × means +, ÷ means –, – means × and + means ÷, then $8 \times 7 - 8 + 40 \div 2 = ?$
   a. 1
   b. $7\dfrac{2}{5}$
   c. $8\dfrac{3}{5}$
   d. 44

12. If '÷' means '×' ; '–' means '×' ; '+' means '÷' and '×' means '–', then $20 \div 12 \times 4 + 8 - 6 = ?$
   a. $8\dfrac{2}{3}$ b. 237
   c. 32   d. 26

13. If '+' means 'divided by' '–' means 'added to' '×' means 'subtracted from' and ÷ means 'multiplied by' then what is the value of $24 \div 12 - 18 + 9$?
   a. –25
   b. 0.72
   c. 15.30
   d. 290

Venn Diagrams

Concept

- All the data should be similar.
- Fill the common area (intersection) first.
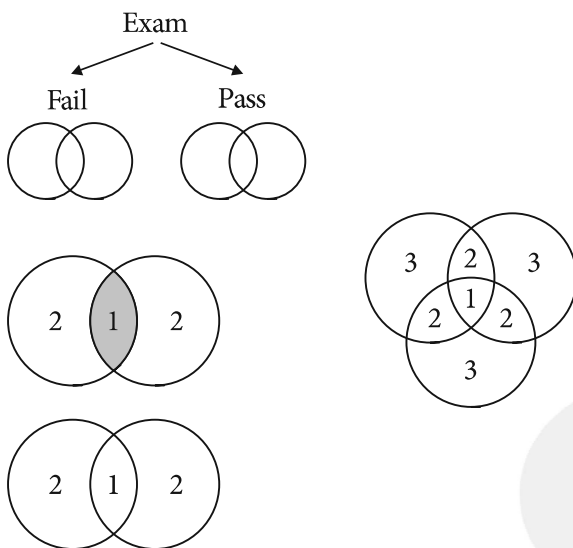
Ex 1. In an examination, 10% of the students failed in Maths, 20% failed in English and 5% failed in both. Find the percentage of students who passed in both the subjects.

Sol:

Maths 5% + 5% + English 15% = 25% failed students
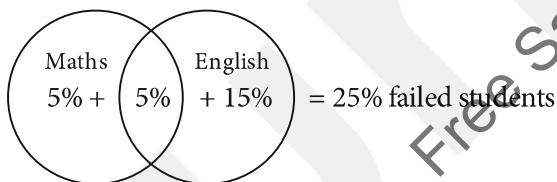
Hence, the percentage of students who passed in both the subjects = 100 – 25 = 75%

Ex 2. In an examination, 45% of the students failed in Maths, 30% failed in English and 15% failed in both. Find the percentage of students who passed in both the subjects.

Sol:

Maths 30% + 15% + English 15% = 60% failed students
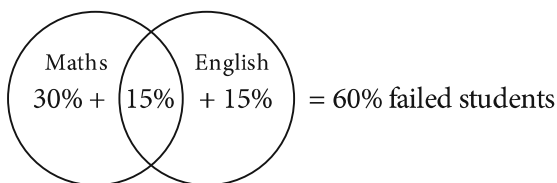
Hence, the percentage of students who passed in both the subjects = 100 – 60 = 40%

Practice Set

1. In an examination, 35% of total students failed in Hindi, 45% failed in English and 20% in both. Find the percentage of those who passed in both the subjects.

   a. 40%   b. 35%

   c. 45%   d. 50%

2. In an examination, 80% of the students passed in English, 85% in Mathematics and 75% in both English and Mathematics. If 40 students failed in both the subjects, find the total number of students.

   a. 350   b. 200

   c. 450   d. 400

3. In an examination, 65% students passed in Civics and 60% in History, 40% passed in both of these subjects. If 90 students failed in History and Civics both, then what is the total number of students?

   a. 600   b. 650

   c. 700   d. 750

4. In an examination, 35% candidates failed in one subject and 42% failed in another subject while 15% failed in both the subjects. If 2500 candidates appeared at the examination, how many passed in either subject but not in both?

   a. 325   b. 1175

   c. 2125   d. None of these

5. In an examination, 34% of the students failed in Mathematics and 42% failed in English. If 20% of the students failed in both the subjects, then the percentage of students who passed in both the subjects was :

   a. 44   b. 50

   c. 54   d. 56

6. 40% of the people read newspaper X, 50% read newspaper Y and 10% read both the papers. What percentage of the people read neither newspaper?

   a. 10%   b. 15%

   c. 20%   d. 25%

7. Out of 450 students of a school, 325 play football, 175 play cricket and 50 neither play football nor cricket. How many students play both football and cricket?

   a. 50   b. 75

   c. 100   d. 225

8. In a hotel, 60% had vegetarian lunch while 30% had non-vegetarian lunch and 15% had both types of lunch. If 96 people were present, how many did not eat either type of lunch?

   a. 20

   b. 24

   c. 26

   d. 28

9. There are 600 boys in a hostel. Each plays either hockey or football or both. If 75% play hockey and 45% play football, how many play both?

   a. 48

   b. 60

   c. 80

   d. 120

10. In a certain office, 72% of the workers prefer tea and 44% prefer coffee. If each of them prefers tea or coffee and 40 like both, the total number of workers in the office is.

    a. 200

    b. 240

    c. 250

    d. 320

11. In a class some students play cricket only, some other students play football only and remaining $1/6^{th}$ students play both cricket and football. Which of the following statements is definitely true ?

    a. Two-thirds of the students play cricket.

    b. Three-fourths of the students play football only.

    c. One-thirds of the students play football only.

    d. None of these

12. In the given figure in a garden, square represent the area where jackfruit trees are grown, circle represent mango trees and triangle represent coconut trees. Which number represent the common area in which all types of trees are grown.

    a. 4   b. 3
    c. 7   d. 8

Previous Year Questions

01. There are 100 students in a particular class. 60% students play cricket, 30% student play football and 10% students play both the games. What is the number of students who play neither cricket nor football?
    (a) 25
    (b) 20
    (c) 18
    (d) 15 **[CSAT 2011]**

02. In the below figure, circle P represents hardworking people, circle Q represents intelligent people, circle R represents truthful people, and circle S represents honest people. Which region represents the people who are intelligent, honest and truthful but not hardworking?
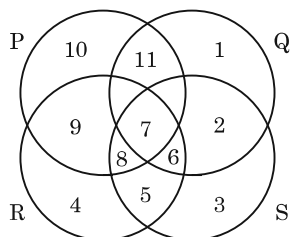
    (a) 6
    (b) 7
    (c) 8
    (d) 11 **[CSAT 2012]**

03. Out of 120 applications for a post, 70 are male and 80 have a driver's license. What is the ratio between the minimum to maximum number of males having driver's license?
    (a) 1 to 2
    (b) 2 to 3
    (c) 3 to 7
    (d) 5 to 7 **[CSAT 2013]**

04. There are 50 students admitted to a nursery class. Some students can speak only English and some can speak only Hind. 10 student can speak both English and Hindi. If the number of students who can speak English 21, then how many students can speak Hindi how many can speak only Hindi and how many can speak English?
    (a) 21, 11 and 29 respectively.
    (b) 28, 18 and 22 respectively.
    (c) 37, 27 and 13 respectively.
    (d) 39, 29 and 11 respectively. **[CSAT 2014]**

05. In a town, 45% population read magazine A, 55% read magazine B, 40% read magazine C, 30% read magazines A and B, 15% read magazines B and C, 25% read magazines A and C; and 10% read all the three magazines. What percentage do *not* read any magazine?
    (a) 10%   (b) 15%
    (c) 20%   (d) 25%   **[CSAT 2015]**

06. Out of 130 students appearing in an examination, 62 failed in English, 52 failed in Mathematics, whereas 24 failed in both English and Mathematics. The number of students who passed finally is
    (a) 40   (b) 50
    (c) 55   (d) 60   **[CSAT 2015]**

07. In a group of persons travelling in a bus, 6 persons can speak Tamil, 15 can speak Hindi and 6 can speak Gujarati. In that group none can speak any other language. If 2 persons in the group can speak two languages only and one person can speak all the three languages, then how many persons are there in the group?
    (a) 21   (b) 22
    (c) 23   (d) 24   **[CSAT 2015]**

Directions: If a square sheet of paper is folded two times from the centre and cuts are made as shown in the problem figure how will it appear when it is opened ?

Select the appropriate figure from the answer choices marked (a), (b), (c) and (d).
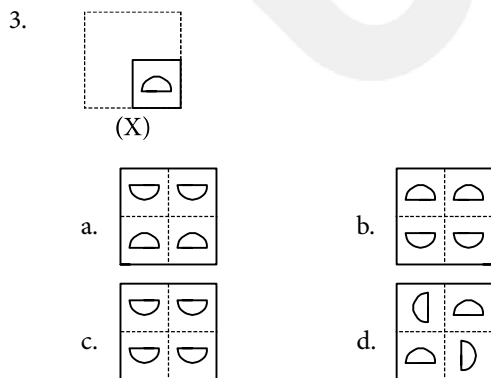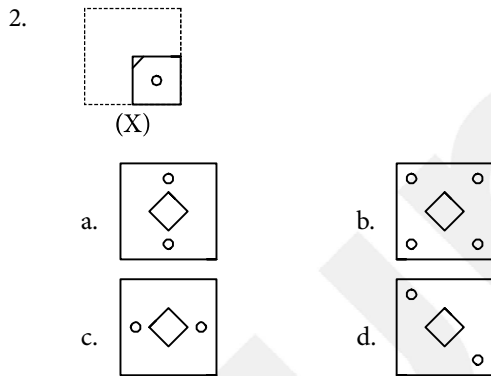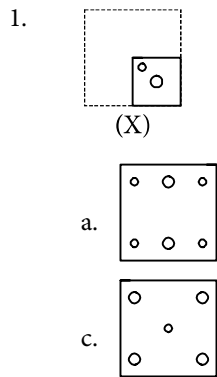
Directions: In each of the following questions a set of three figures A, B and C showing a sequence of folding of a piece of paper. Figure (C) shows the manner in which the folded paper has been cut. These three figures are following by four answer figures from which you have to choose a figure which would most closely resemble the unfolded from of figure (C).

1.

(X)

a.
b.
c.
d.

2.

(X)

a.
b.
c.
d.

3.

(X)

a.
b.
c.
d.

4.

(A)   (B)   (C)

a.
b.
c.
d.

5.

(A)   (B)   (C)

a.
b.
c.
d.

6.

(A)   (B)   (C)

a.
b.
c.
d.

Chapter 28

Mirror & water images
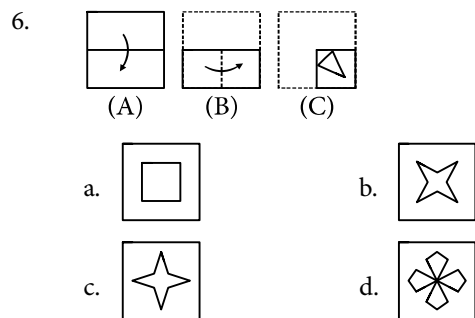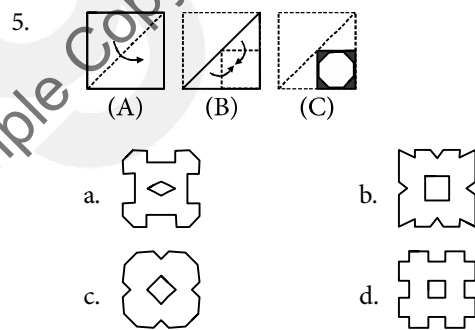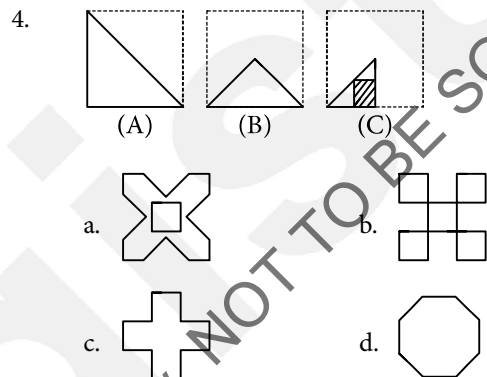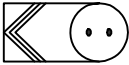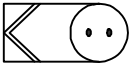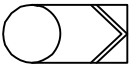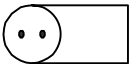
Directions (1-2): Find the correct option for the mirror images for the following questions.

1.
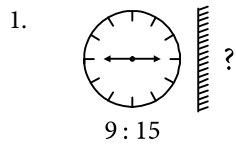
9 : 15

a. 2 : 45   b. 3 : 15
c. 9 : 15   d. 9 : 45

2.
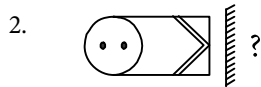
a.

b.

c.

d.

Directions (3-5): In each of the following questions, you are given a combination of alphabets and/or numbers followed by alternatives (a), (b), (c) and (d). Choose the alternative which most closely resembles the mirror image of the given combination.

3.   N A T I O N A L

   a.   ⅃AИOITAИ
   b.   ⅃AИOITAИ
   c.   ⅃AИOITAИ
   d.   ⅃AИOITAИ

4.   1 9 6 5 I N D O P A K

   a.   ⋊AꟼO♄ИI2961
   b.   ⋊AꟼO♄И I O961
   c.   ⋊AꟼO♄И I2961
   d.   ⋊AꟼO♄И I2961

5.   T E R M I N A T E

   a.   TƎꓳꟼMIИATƎ
   b.   ƎTAИIMꓤƎT
   c.   ƎTAИIMꓤƎT
   d.   ETAИIMꓤƎT

Directions (6-10): In each of the following questions, choose the correct mirror-image of the figure (X) from amongst the four alternatives (a), (b), (c) and (d) given along with it.

6.

(X)

a.

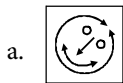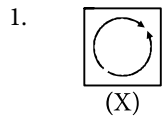

b.

c.

d.

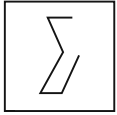7.

(X)

a.

b.

c.

d.

8.

M_1

M_2
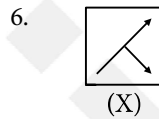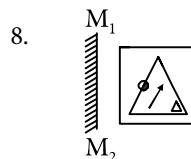
a.

b.

c.

d.

Directions (1-10): In each of the following questions, you are given a figure (X) followed by four alternative figures (a), (b), (c) and (d) such that figure (X) is embedded in one of them. Trace out the alternative figure which contains figure (X) as its part.

1.

(X)

a.   b.

c.   d.

2.

(X)

a.   b.

c.   d.

3.

(X)

a.   b.

c.   d.

4.

(X)

a.   b.

c.   d.

5.

(X)

a.   b.

c.   d.

6.

(X)

a.

b.

c.

d.
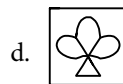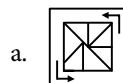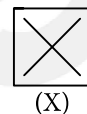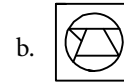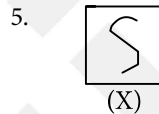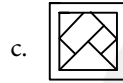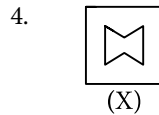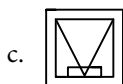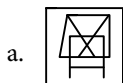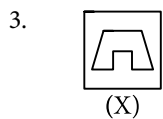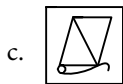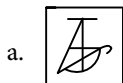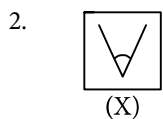
01. Examine the following figure:

Which one of the following figures has the above figure embedded in it?

(a)   (b)

(c)   (d)

[CSAT 2014]

ANSWER KEY

01. (c)

Chapter **30** Seating arrangements
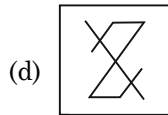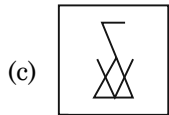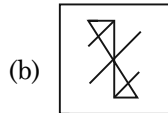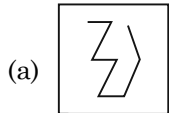
Directions (1 to 5): Study the following information carefully and answer the questions given below.

A, B, C, D, E, F, G, H and K are sitting around a circle facing the centre. B is fourth to the left of G who is second to the right of C. F is fourth to the right of C and second to the left of K. A is fourth to the right of K. D is not immediate neighbour of either K or B, H is third to the right of E.

1. Who is fourth to the right of D?
   a. K   b. H
   c. E   d. B
   e. None of these

2. In which of the following combinations is the third person sitting in between the first and second persons?
   a. EKB   b. CHB
   c. AGC   d. FGD
   e. None of these

3. Who is third to the right of H?
   a. A   b. D
   c. G   d. F
   e. None of these

4. Who is the fourth to the left of E?
   a. A   b. c
   c. G   d. Data inadequate
   e. None of these

5. Who is second to the right of K?
   a. C   b. H
   c. F   d. E
   e. Data inadequate

Directions (6 to 10): Study the following information carefully to answer these questions.

Eight friends A, B, C, D, E, F, G and H are sitting around a circle facing the centre. B is third to the right of A, who is third to the right of C. F is second to the right of E who is not an immediate neighbour of B. D sits second to the left of H, who sits second to the left of G.

6. Who sits second of the left of A?
   a. G   b. D
   c. H   d. E
   e. None of these

7. Who sits to the immediate left of B?
   a. G   b. F
   c. D   d. H
   e. None of these

8. What is the position of H with respect to F?
   a. To the immediate right
   b. Third to the left
   c. Second to the left
   d. Second to the right
   e. Third to the right

9. Four of the following five are alike in a certain way based on their positions in the given arrangement and so form a group. Which is the one that does not belong to the group?
   a. AD   b. HG
   c. EF   d. BF
   e. CE

10. Which of the following pairs has the first person sitting to the immediate right of the second person?
    a. BH   b. DF
    c. CF   d. EA
    e. None of these

Directions (11 to 15): Study the following information and answer these questions.

1. A, B, C, D, E, F, G and Hare eight friends sitting in a circle facing the centre.

2. F is on the immediate right of D while B is on the immediate left of E.

3. A is sitting opposite neither C nor H. And A is not the neighbour of H.

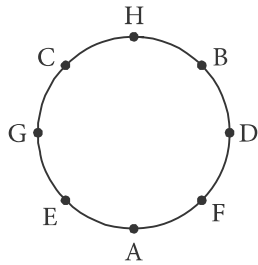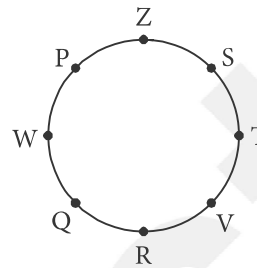4. E and H are sitting adjacent to C.

Solutions :

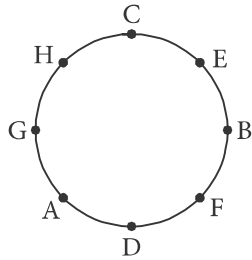**Questions (1 to 5):**
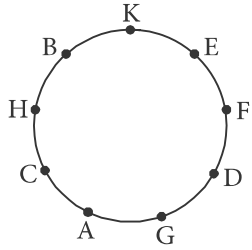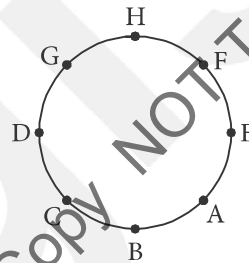

Questions (16 to 20):
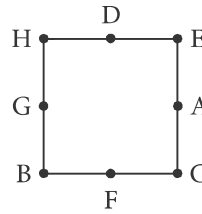
Questions (6 to 10):

Questions (21 to 25):

Questions (11 to 15):

Questions (26 to 30):

Previous Year Questions

01. Five people A, B, C, D and E are, seated about a round table. Every chair is spaced equidistant from adjacent chairs.

    (i) C is seated next to A.

    (ii) A is seated two seats from D.

    (iii) B is not seated next to A.

    Which of the following must be true?

    1. D is seated next to B.

    2. E is seated next to A.

    Select the correct answer from the codes given below:

    (a) 1 only

    (b) 2 only

    (c) Both 1 and 2

    (d) Neither 1 nor 2   **[CSAT 2013]**

02. Four children are sitting in a row. A is occupying the seat next to B but not next to C. If C is not sitting next to D, who is/are occupying seat/seats adjacent to D?

    (a) B

    (b) A

    (c) B and A

    (d) Impossible to tell   **[CSAT 2014]**

03. Eight students A, B, C, D, E, F, G and H sit around a circular table, equidistant from each other, facing the centre of the table, not necessarily in the same order. B and D sit neither adjacent to C nor opposite to C. A sits in between E and D, and F sits in between B and H. Which one of the following is definitely correct?

    (a) B sits in between A and G.

    (b) C sits opposite to G.

    (c) E sits opposite to F.

    (d) None of the above   **[CSAT 2022]**

04. Six persons A, B, C, D, E and F are sitting equidistant from each other round a circular table (facing the centre of the table).

    Consider the Question and two Statements given below:

    Question: Who is sitting on the immediate left of A?

    Statement-1: B is sitting opposite to C and D is sitting opposite to E.

    Statement-2: F is sitting on the immediate left of B.

    Which one of the following is correct in respect of the Question and the Statements?

    (a) Statement-1 alone is sufficient to answer the Question.

    (b) Statement-2 alone is sufficient to answer the Question.

    (c) Both Statement-1 and Statement-2 are sufficient to answer the Question.

    (d) Both Statement-1 and Statement-2 are not sufficient to answer the Question.

                                        [CSAT 2022]

ANSWER KEY

01. (c)   02. (b)   03. (d)   04. (d)

Chapter 31 — Data sufficiency

Data sufficiency

Data sufficient means a unique answer.

1. Read the question statement first.
2. Come to statement 1 alone and try to find the answer, if we can find a unique answer then statement 1 alone would be sufficient to find the answer.
3. If statement 1 alone is not sufficient, move to statement 2 and read that statement alone (forget about statement 1) and try to find the answer, "if it can be obtained uniquely, statement 2 alone would be sufficient to find the answer.
4. If from statement 2, we cannot find a unique answer, try to find the answer using both the statements together, in that case both the statements together would be sufficient to find the answer. Else answer cannot be obtained even after using both the statements together.
5. Sometimes questions can be uniquely answered either from statement 1 alone or from statement 2 alone, in that case either one of the two statements alone would be sufficient to find the answer.
6. If a question can be answered using any one statement alone and using both the statements together, in that situation the answer from the single statement would be preferred.

1. Who is tallest among P, Q, R, S & T?
   I. S is shorter than Q. P is shorter than only T.
   II. Q is taller than only S. T is taller than P and R.
   Options :
   1. Statement I alone is sufficient to answer the question.
   2. Statement II alone is sufficient to answer the question.
   3. Either statement I alone or statement II alone is sufficient to answer the question.
   4. Both statements I & II together are necessary to answer the question.

2. How is 'face' written in that code language?
   I. In a certain code language, 'no one with face' is coded as 'fo to om sop' and 'no one has face' is coded as 'om sit fo sop'.
   II. In a certain code language, 'face of no light' is coded as 'om mot fo kiz' and 'no one is smart' is coded as 'sop fo sip lik'.
   Options :
   1. Statement I alone is sufficient to answer the question.
   2. Statement II alone is sufficient to answer the question.
   3. Both statements I & II together are not sufficient to answer the question.
   4. Both statements I & II together are necessary to answer the question.

3. What is Monica's position with respect to Rahul?
   I. In a row of 25 students, Monica is sitting 12th from right end of row and Rahul is sitting 20th from left end of the row.
   II. Monica is 4th from right end and Rahul is 8th from left end.
   Options :
   1. Statement I alone is sufficient to answer the question.
   2. Statement II alone is sufficient to answer the question.
   3. Both statements I & II together are not sufficient to answer the question.
   4. Both statements I & II together are necessary to answer the question.

4. Who has secured least marks among P, Q, R , S & T ?
   I. S has secured less marks than only R and T.
   II. Q secured more marks than P.
   Options :
   1. Statement I alone is sufficient.
   2. Statement II alone is sufficient.
   3. Both statements I & II together are not sufficient.
   4. Both statements I & II together are necessary.

5. On which floor is Shikha residing?
   I. In a six storey building (Ground floor is parking space), Rekha is on fourth floor. Shikha likes to reside only on even numbered floors. Reema is not on the topmost floor.
   II. Reema is two floors below Peter who is 3 floors above Shikha.
   Options :
   1. Statement I alone is sufficient.
   2. Statement II alone is sufficient.
   3. Both statements I & II together are not sufficient.
   4. Both statements I & II together are necessary.

Practice Set

1. How is GREEN written in a code language ?
   I. GREEN AND BLACK is coded as '#@7' and ORANGE AND PINK is coded as '$%#'.
   II. PINK AND RED is coded as '#$8' and YELLOW AND GREEN is coded as '6@#'.
   Options :
   1. Statement I alone is sufficient.
   2. Statement II alone is sufficient.
   3. Both statements I & II together are not sufficient.
   4. Both statements I & II together are necessary.

2. In which month did Rahul go to Kanpur for business ?
   I. Rahul's son remembers that he went after 20th August but before 10th September.
   II. Varun, friend of Rahul remembers that he went Kanpur in the 3rd quarter of the fiscal year.
   Options :
   1. Statement I alone is sufficient to answer the question.
   2. Statement II alone is sufficient to answer the question.
   3. Both statements I & II together are not sufficient to answer the question.
   4. Both statements I & II together are necessary to answer the question.

3. Madan's flat is on which floor of 5 floor apartments ?
   I. Harish flat, which is exactly above to Madan, is exactly below Karan's flat which is on fifth floor.
   II. Madan's flat is exactly above Gopal's flat, whose flat is exactly above Nitin's first floor flat.
   Options :
   1. Statement I alone is sufficient to answer the question.
   2. Statement II alone is sufficient to answer the question.
   3. Both statements I & II together are not sufficient to answer the question.
   4. Either statement I or II alone is sufficient to answer the question.

4. How many pencils does the shopkeeper sells on Sunday?
   I. On Sunday he sold 12 more pencils than he sold the previous day.
   II. He sold 28 pencils each on Thursday and Saturday.
   Options :
   1. Statement I alone is sufficient to answer the question.
   2. Statement II alone is sufficient to answer the question.

3. Both statements I & II together are not sufficient to answer the question.
4. Both statements I & II together are necessary to answer the question.

5. How many boys students are there in the class ?
   I. 65% girls students are there in the class.
   II. The no. of boys students is half that of girls.
   Options :
   1. Statement I alone is sufficient to answer the question.
   2. Statement II alone is sufficient to answer the question.
   3. Both statements I & II together are not sufficient to answer the question.
   4. Both statements I & II together are necessary to answer the question.

6. Which of the five trains A, B, C, D and E is the best ?
   I. Train D is better than train E , A and C but not as good as train B.
   II. Train D is better than train C but not as good as train B which is better than train E.
   Options :
   1. Statement I alone is sufficient to answer the question.
   2. Statement II alone is sufficient to answer the question.
   3. Both statements I & II together are not sufficient to answer the question.
   4. Both statements I & II together are necessary to answer the question.

7. On which date in April is definitely Arun's father's birthday ?
   I. Arun correctly remembers that his father's birthday is before 22nd April but after 14th April.
   II. Arun's brother correctly remembers that their father's birthday is after 19th April but before 28th April.
   Options :
   1. Statement I alone is sufficient to answer the question.
   2. Statement II alone is sufficient to answer the question.
   3. Both statements I & II together are not sufficient to answer the question.
   4. Both statements I & II together are necessary to answer the question.

Chapter 32 — Syllogism

1. *Statements*: All tubes are handles. All cups are handles.
   Conclusions: I. All cups are tubes.
   II. Some handles are not cups.
   a. If only conclusion I follows.
   b. If only conclusion II follows.
   c. If either conclusion I or II follows.
   d. If neither conclusion I nor II follows.

2. *Statements*: Some dedicated souls are angles.
   All social workers are angles.
   Conclusions: I. Some dedicated souls are social workers.
   II. Some social workers are dedicated souls.
   a. If only conclusion I follows.
   b. If only conclusion I follows.
   c. If either conclusion I or II follows.
   d. If neither conclusion I nor II follows.

3. *Statements*: All roads are poles. No pole is a house.
   Conclusions: I. Some roads are houses.
   II. Some houses are poles.
   a. If only conclusion I follows.
   b. If only conclusion I follows.
   c. If either conclusion I or II follows.
   d. If neither conclusion I nor II follows.

4. *Statements*: Many scooters are trucks.
   All trucks are trains.
   Conclusions: I. Some scooters are trains.
   II. No truck is a scooter.
   a. If only conclusion I follows.
   b. If only conclusion I follows.
   c. If either conclusion I or II follows.
   d. If neither conclusion I nor II follows.

5. *Statements*: Some papers are pens.
   Angle is a paper.
   Conclusions: I. Angle is not a pen.
   II. Angle is a pen.
   a. If only conclusion I follows.
   b. If only conclusion I follows.
   c. If either conclusion I or II follows.
   d. If neither conclusion I nor II follows.

6. *Statements*: All shares are debentures.
   No debenture is an equity.
   Conclusions: I. No equity is a share.
   II. Some debentures are shares.
   III. No share is an equity.
   a. Only I follows
   b. Only II follows
   c. Only III follows
   d. All follow

7. *Statements*: No box is toy. All toys are blocks.
   Conclusions: I. Some blocks are toys.
   II. Some blocks are boxes.
   III. No block is box.
   a. Only I follows
   b. Only either II or III follows
   c. Only either II or III, and I follow
   d. None follows

Directions (Questions 8 to 12): Each question given below consists of five or six statements followed by options consisting of three statements put together in a specific order. Choose the option which indicates a valid argument containing logically related statements that is, where the third statement is a conclusion drawn from the preceding two statements.

8. A : All balls are locks.
   B : All keys are locks.
   C : All keys are balls.
   D : Some keys are locks.
   E : Some locks are balls.
   F : No ball is lock.
   a. ACD   b. BEF
   c. CDE   d. CEF

Previous Year Questions

Directions for the following 2 (two) items: Each of the following two items consists of four statements. Of these four statements, two cannot both be true, but both can be false. Study the statements carefully and identify the two that satisfy the above condition. Select the *correct* answer using the codes given below each set of statements:

01. Examine the following statements:
    1. All animals are carnivorous.
    2. Some animals are not carnivorous.
    3. Animals are not carnivorous.
    4. Some animals are carnivorous.
    (a) 1 and 3
    (b) 1 and 2
    (c) 2 and 3
    (d) 3 and 4   **[CSAT 2011]**

02. Examine the following statements:
    1. All trains are run by diesel engine.
    2. Some trains are run by diesel engine.
    3. No train is run by diesel engine.
    4. Some trains are not run by diesel engine.
    (a) 1 and 2
    (b) 2 and 3
    (c) 1 and 3
    (d) 1 and 4   **[CSAT 2011]**

03. Consider the following statements:
    1. All artists are whimsical.
    2. Some artists are drug addicts.
    3. Frustrated people are prone to become drug addicts.
    From the above three statements it may be concluded that
    (a) Artists are frustrated.   **[CSAT 2012]**
    (b) Some drug addicts are whimsical.
    (c) All frustrated people are drug addicts.
    (d) Whimsical people are generally frustrated.

04. Consider the following statements followed by two conclusions:
    Statements: Some men are great.
                   Some men are wise.

Conclusion I: Men are either great or wise.
Conclusion II: Some men are neither great nor wise.
Which one of the following is correct?
(a) Only conclusion I is valid.
(b) Only conclusion II is valid.
(c) Both the conclusions are valid.
(d) Neither of the conclusion is valid.

                                        [CSAT 2015]

05. Examine the following statements:
    1. All colours are pleasant.
    2. Some colours are pleasant.
    3. No colour is pleasant.
    4. Some colours are not pleasant.
    Given that statement 4 is true, what can be definitely concluded?
    (a) 1 and 2 are true
    (b) 3 is true
    (c) 2 is false
    (d) 1 is false

                                        [CSAT 2017]

06. Consider the following Statements and Conclusions:
    Statements-1:   Some rats are cats.
    Statements-2:   Some cats are dogs.
    Statements-3:   No dog is a cow.
    Conclusions-I: No cow is a cat.
    Conclusions-II: No dog is a rat.
    Conclusions-III: Some cats are rats.
    Which of the above conclusions is/are drawn from the statements?
    (a) I, II and III
    (b) Only I and II
    (c) Only III
    (d) Only II and III

                                        [CSAT 2019]

Chapter 33 — Puzzles

Directions (Questions 1 to 3): There are 4 persons A, B, C, D. They are going to Hyderabad, Kanpur, Bhopal and Yamuna Nagar. They have 4 bikes i.e. Kinetic, Yamaha, Honda and Bajaj but not in the given order.

. No two initials match.

. A has Yamaha. B goes to Hyderabad. D has Bajaj.

. A person who has Honda going to Kanpur.

1. Who is going to Bhopal?
   a. A   b. B
   c. C   d. D

2. Bajaj is owned by
   a. A   b. B
   c. C   d. D

3. A person who has Kinetic, going to
   a. Kanpur   b. Hyderabad
   c. Bhopal   d. Yamuna Nagar

4. During an evening party, when Ms. Black, Ms. Brown and Ms. White met, Ms. Brown remarked, "It is interesting that our dresses are white, black or brown, but for each of us the name does not match the colour of the dress!" Ms. White replied, "But your white dress does not suit you!". Pick the correct answer.
   a. Ms. White's dress was brown.
   b. Ms. Black's dress was white.
   c. Ms. White's dress was black.
   d. Ms. Black's dress was black.

Directions (Questions 5 to 8): Read the information given below and answer the questions that follow:

Four young men Raj, Prem, Ved and Ashok are friendly with four girls Sushma, Kusum, Vimla and Poonam. Sushma and Vimla are friends. Ved's girl friend does not like Sushma and Vimla. Kusum does not care for Ved. Prem's girl friend is friendly with Sushma. Sushma does not like Raj.

5. Who is Raj's girl friend?
   a. Sushma
   b. Kusum
   c. Vimla
   d. Poonam

6. With whom is Sushma friendly?
   a. Raj   b. Prem
   c. Ved   d. Ashok

7. Who is Poonam's boy friend?
   a. Ashok   b. Ved
   c. Prem   d. Raj

8. Who does not like Sushma and Vimla?
   a. Poonam   b. Raj
   c. Ashok   d. Ved

9. Three persons A, B and C wore shirts of black, blue and orange colours (not necessarily in that order) and pants of green, yellow and orange colours (not necessarily in that order). No person wore pant and shirt of the same colour. Further, it is given that

   1. A did not wear shirt of black colour.
   2. B did not wear shirt of blue colour.
   3. C did not wear shirt of orange colour.
   4. A did not wear pant of green colour.
   5. B wore pant of orange colour.

   What were the colours of pant and shirt worn by C, respectively?
   a. Yellow and Black   b. Yellow and Blue
   c. Green and Blue   d. Orange and Black

Directions (Questions 10 to 13): Study the following information carefully and answer the given questions:

A Business School with six Professors L, M, N, O, P and Q has decided to implement a new scheme of course management. Each Professor has to coordinate one course and support another course. This semester, O's support course is Finance, whole three others have it in coordinator's role. P and Q have Marketing as one of their subjects. Q coordinates Operations, which is a support course for both N and P. Finance and It are L's subjects. Both L and O have same subjects. Strategy is a support course for only one of the Professors.

10. Who coordinates the IT course?
    a. L   b. N
    c. O   d. None of these

80. Seven books P, Q, R, S, T, U and V are placed side by side. R, Q and T have blue covers and other books have red covers. Only S and U are new books and the rest are old. P, R and S are law reports; the rest are Gazetteers. Books of old Gazetteers with blue covers are

(a) Q and R
(b) Q and U
(c) Q and T
(d) T and U

[CSAT 2021]

81. The digits 1 to 9 are arranged in three rows in such a way that each row contains three digits, and the number formed in the second row is twice the number formed in the first row; and the number formed in the third row is thrice the number formed in the first row. Repetition of digits is not allowed. If only three of the four digits 2, 3, 7 and 9 are allowed to use in the first row, how many such combinations are possible to be arranged in the three rows?

(a) 40
(b) 3
(c) 2
(d) 1   **[CSAT 2022]**

ANSWER KEY

01. (c)	02. (a)	03. (b)	04. (a)	05. (a)	06. (b)	07. (c)	08. (c)	09. (b)	10. (d)
11. (b)	12. (c)	13. (c)	14. (a)	15. (d)	16. (a)	17. (b)	18. (d)	19. (a)	20. (b)
21. (c)	22. (b)	23. (c)	24. (d)	25. (a)	26. (b)	27. (d)	28. (c)	29. (b)	30. (c)
31. (b)	32. (a)	33. (b)	34. (d)	35. (d)	36. (a)	37. (c)	38. (b)	39. (b)	40. (c)
41. (d)	42. (b)	43. (b)	44. (a)	45. (c)	46. (b)	47. (b)	48. (c)	49. (a)	50. (a)
51. (b)	52. (b)	53. (c)	54. (a)	55. (b)	56. (a)	57. (c)	58. (b)	59. (c)	60. (b)
61. (c)	62. (a)	63. (b)	64. (b)	65. (b)	66. (b)	67. (d)	68. (c)	69. (b)	70. (a)
71. (b)	72. (a)	73. (d)	74. (a)	75. (a)	76. (c)	77. (d)	78. (b)	79. (b)	80. (c)
81. (c)									

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✈ t.me/**UNGIST**

▶ YouTube.com/**@UNGISTIAS**

✉ ungistsolutions@gmail.com

ungist